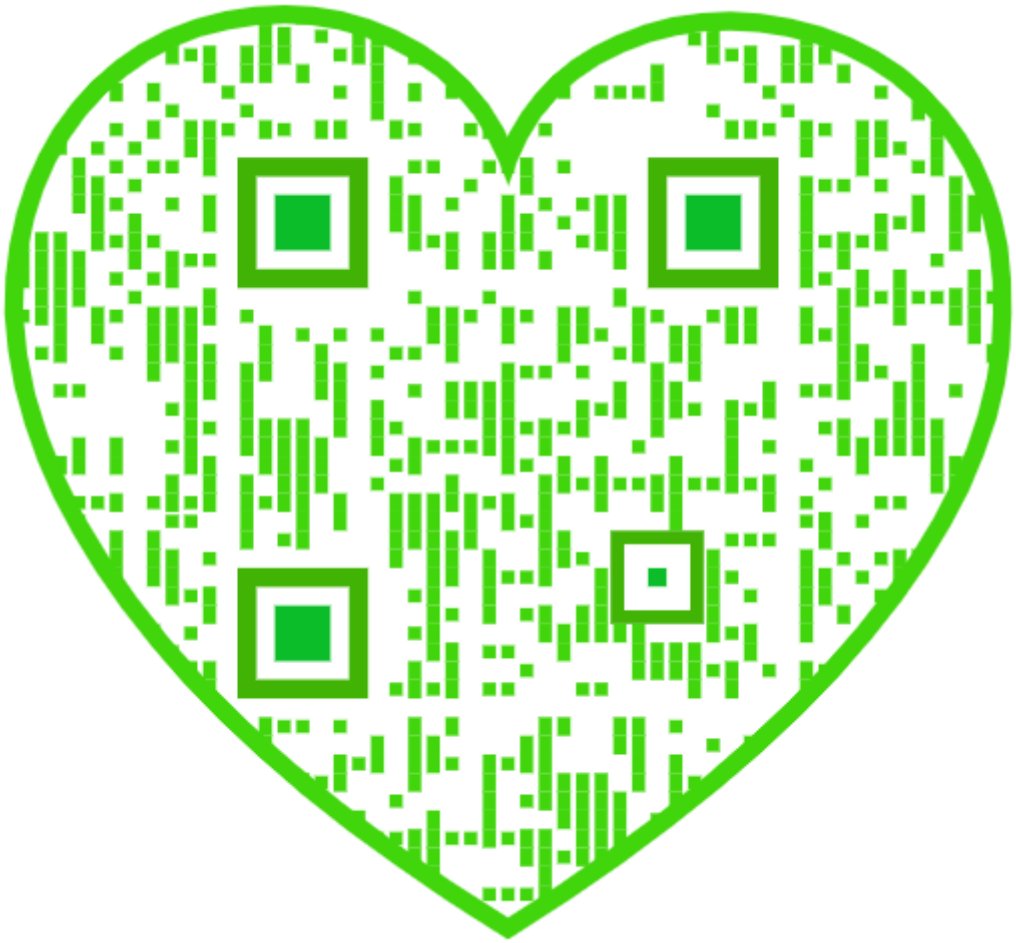

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ARITHMETIC

OF

Magnetism

∴ AND ∴

Electricity.

BY JOHN T. MORROW, M. E.,

— AND —

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PREFACE.

It has been the endeavor of the authors to keep rigidly within the scope outlined by the title of this book. There has been no attempt at explanation of the phenomena involved and no deduction of the rules given, although in some cases some of the more obvious deductions *from* the rules have been indicated.

The object has been to enumerate all those rules of electricity and magnetism which are directly connected with their commercial applications and to give numerical examples of each.

The phenomena of magnetism have, we believe, been treated with more completeness and detail here than has been attempted in previons books of this character, and the design of dynamos has been treated in accordance with the latest and best practice of working engineers.

The general laws of electric circuits are first taken up and developed at length. This is followed by a chapter on batteries, after which the general subject of magnetism is treated.

Finally, the application of the laws in the design of electro-magnetic machinery is worked out, both for direct and for alternating currents.

An Appendix on the electric railway is added, since several of the laws of physics, neither electric nor magnetic are involved, which are rather necessary to a clear understanding of the electro-magnetic part of the system.

JOHN T. MORROW.
THORBURN REID.

Aug. 1st, 1893.

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ARITHMETIC

— OF —

Magnetism and Electricity.

CHAPTER I.

ELECTROMOTIVE FORCE, CURRENT AND RESISTANCE.

The principal units employed by practical electricians are:—

The *Ampere*, or unit of current.

The *Volt*, or unit of electromotive force.

The *Ohm*, or unit of electric resistance.

These three units are based upon certain abstract units derived by mathematical reasoning and experimentally proven laws from the three fundamental units: —

The *Centimetre*, or unit of length.

The *Gramme*, or unit of mass.

The *Second*, or unit of time.

The system of "*Absolute Units*" derived from these is often denominated the "C. G. S." system

of units, to distinguish it from other systems based on other fundamental units.

Every system of measurement is based upon some experimental fact or law.

We can only measure an electric current by the effects it produces.

An electric current can

First: — Deposit metals from their chemical solutions.

Second: — Heat the conductor through which it flows.

Third: — Attract or repel a neighboring parallel current.

Fourth: — Accumulate as an electric charge which can repel or attract a neighboring charge of electricity.

Fifth: — Produce in its neighborhood a magnetic field, that is to say, it can exert a force upon a magnetic pole placed near it.

The *last*, or *fifth*, of the foregoing effects is made the basis of the system now adopted, and is the best for practical purposes.

The Absolute Unit of Current is one of such a strength that when one centimetre length of its circuit is bent into an arc of one centimetre radius, the current in it exerts a force of one dyne on a unit magnet-pole placed at the centre.

The Unit of Magnetism, or Unit Magnet Pole, is one of such strength that when placed at a dis-

tance of one centimetre (in air) from a similar pole of equal strength it repels it with a force of one dyne.

The Unit Magnet Pole may be further described as such a pole that at a distance of one centimetre therefrom, there is one line of induction per square centimetre which may be called the *Unit Line of Induction.*

The Absolute Unit of Force, i. e. *the Dyne or C. G. S. unit of force*, is that force which, if it act for a second on one gramme, gives to it a velocity of one centimetre per second.

Now a current strong enough to fulfill the above definition of the *Absolute Unit of Current* is thought to be ten times too great for practical use.

Accordingly: *The practical unit of current is fixed at one tenth part of the absolute unit and is called "One Ampere."*

It may further be noted that a current of one ampere strength will cause the deposition in one hour of 1.174 grammes (18.116 grains) of copper in a copper electrolytic cell.

Now, if a conductor be made to cut a *unit line of induction* at the velocity of one centimetre per second an electromotive force will be generated equal to one one hundred millionth ($\frac{1}{100,000,000}$), of that electromotive force which is called a *Volt.*

Or, the Volt, is that electromotive force which would be generated by a conductor cutting across

100,000,000 C. G. S. lines of Inductions per second.

The current produced by a given electromotive force depends upon the resistance opposed to its flow and varies according to the nature of the substance through which it passes; the current being *less*, as the resistance is *greater*, in accordance with the famous law discovered by Dr. Ohm.

Ohm's law states that the current is directly proportional to the electromotive force that is exerted in the circuit and is inversely proportional to the resistance of the circuit.

This law is more concisely expressed as follows:

$$\text{Current} = \frac{\text{Electromotive force}}{\text{Resistance.}}$$

Let E Stand for the number of units of electromotive force; R for the number of units of Resistance of the circuit, and C for the current that results.

The above expression can then be written:

$$C = \frac{E}{R}$$

Now let C = *one absolute unit of current*, E = *one absolute unit of electromotive force*, then R = *one absolute unit of resistance*, i. e.,

$$\frac{\text{One absolute unit of Electromotive force}}{\text{One absolute unit of Current}} = \text{One absolute unit of Resistance.}$$

But as we have already practical units of *current* and *electromotive force* we must have a corresponding practical unit of *resistance*.

Substituting the practical unit of current and electromotive force in the above, we have

$$\frac{\text{100,000,000 C. G. S. units of E. M. F.,}}{\text{.1 C. G. S. unit of current.}} = \text{1000,000,000 C. S. G. units of Resistance.}$$

Our *practical unit of resistance* then to be used with the *Volt* and *Ampere is one thousand million*, (*1000,000,000,000*) *C. G. S. units of resistance* and is called the *Ohm.*

The accepted physical *value of the ohm* is a resistance equal to that of a column of mercury one square millimetre (.00155 sq. in.) in cross section and 106 centimetres (41.7322 in.) long.

Ohm's Law may be expressed in three ways, as is illustrated by Rule 1, 2 and 3.

Hereafter the abbreviation E. M. F. will be used for Electromotive force.

Rule 1. Current equals E. M. F. divided by Resistance, i. e.

$$C = \frac{E}{R}$$

EXAMPLE 1.

The resistance of a circuit was found to be 4 ohms, the E. M. F. was 110 volts; required, the current in amperes.

Solution: By Rule 1 we have $C = \frac{E}{R}$ or $C = 110 \div 4 = 27.5$ amperes. Ans.

EXAMPLE 2.

The resistance of a certain circuit is 10,000 ohms, the E. M. F. is 125 Volts. Find the current in Amperes.

Solution: From Rule 1 we have $C = \frac{E}{R} = \frac{125}{10000} = .0125$ amperes. Ans.

Rule 2. Electromotive force equals current multiplied by Resistance, or $E = C \times R$.

EXAMPLE 3.

A battery having a resistance of 2 ohms sends a current of .03 amperes through a conductor whose resistance is 48 ohms, what is the E. M. F. of the battery?

Solution: The total resistance of the circuit through which the current of .03 ampere flows is the sum of the *External* resistance, 45 ohms, and the Internal resistance, 2 ohms, or $45 + 2 = 47$ ohms.

By Rule 2 we have $E = C \times R$, or $E = 47 \times .03 = 1.41$ Volts. Ans.

EXAMPLE 4.

The resistance of a certain circuit is 2.2 ohms; the current is 50 amperes. Find the E. M. F.

Solution: By Rule 2, $E = C \times R = 2.2 \times 50 = 110$ Volts. Ans.

Rule 3. Resistance equals E. M. F. divided by Current.

EXAMPLE 5.

The E. M. F. in the mains of a circuit is found to be 480 Volts, the current is 10 amperes; find the resistance, or $R = \frac{E}{C}$

Solution:

Ans. By Rule 3, $R = \frac{E}{C} = \frac{480}{10} = 48$ ohms.

EXAMPLE 6.

The current required to bring a certain incandescent lamp to candle power was found to be .44 ampere. The E. M. F. at the lamp was 110 Volts. Required the resistance of the lamp.

Solution:

By Rule 3, $R = \frac{E}{C} = \frac{110}{.44} = 250$ ohms. Ans.

CHAPTER II.

GENERAL LAWS OF ELECTRIC CIRCUITS.

Experiment has determined the following laws for resistance to permanent currents in conductors.

Rule 4. The resistance of a conductor is proportional to its length.

Example 1.

If the resistance of one mile of a certain electric light conductor be 4.5 ohms, what is the resistance of 12.9 miles of the same conductor?

Solution by Rule 4. If the resistance of one mile of the conductor is 4.5 ohms, the resistance of 12.9 miles is 12.9 as much as that of one mile i.e., $12.9 \times 4.5 = 58.05$ ohms. Ans.

Example 2.

If the resistance of 2100 feet of a cable be 92 ohms, what is the resistance of 8400 feet?

Solution by Rule 4. If the resistance of 2100 feet $= 92$ ohms, the resistance of one foot is $92 \div 2100 = .000438$ ohms, and the resistance of 8400 feet is $8400 \times .000438 = 3.67$ ohms. Ans.

Example 3.

The resistance of 200 yards of a certain con-

ductor is $5\frac{1}{2}$ ohms, what length of wire has a resistance of 23.25 ohms?

Solution by Rule 4. If $5\frac{1}{2}$ ohms is the resistance of 200 yards, 1 ohm is the resistance of $200 \div 5\frac{1}{2} = 36.363$ yards and 23.25 ohms is the resistance of $23.25 \times 36.36 = 845.37$ yds. Ans.

Example 4.

The resistance of a certain conductor is 3 ohms and the resistance of a mile of the same conductor is found to be 2.4 ohms, what is the length of the conductor?

Solution by Rule 4. If the resistance of one mile be 3 ohms that length whose resistance is 2.4 ohms is $3 \div 2.4 = 1.25$ miles. Ans.

Rule 5. The resistance of a conductor is inversely proportional to the area of its cross section.

Note.—Remember that the areas of circular cross sections vary with the squares of their diameters.

Example 5.

If the resistance of 130 yards of copper wire $\frac{1}{16}''$ in diameter is one ohm, what is the resistance of the same length of copper wire $\frac{1}{8}''$ in diameter?

Solution by Rule 5.

$$\frac{(\frac{1}{16})^2}{(\frac{1}{8})^2} = \frac{R}{1} \text{ where } R = \text{resistance required.}$$

Therefore $R = .25$ ohms. Ans.

Example 6.

What is the resistance of a mile of copper wire which has a diameter of 60 mils, if the resistance of a mile of copper wire 80 mils in diameter is 8.29 ohms?

Note.—A mil is the thousandth part of an inch and the diameters of wires are generally expressed in terms of this unit.

Solution by Rule 5.

$\frac{80^2}{60^2} = \frac{R}{8.29}$ where R = resistance required.

$\therefore R = 80^2 \times 8.29 \div 60^2 = 14.73$ ohms. Ans.

Note.—The symbol $\therefore$ is read therefore.

Example 7.

What is the diameter of a copper wire one mile long which has a resistance of 29 ohms, if a mile of copper wire 70 mils ($\frac{70}{1000}''$) in diameter has a resistance of 10.82 ohms?

Solution by Rule 5. First find the diameter, d, of a wire one mile in length having a resistance of one ohm $\frac{70^2}{d^2} = \frac{1}{10.82}$, or $d^2 = 70^2 \times 10.82 \therefore d = 230$ mils, and the diameter d, of a wire one mile long whose resistance is 29 ohms would be $\frac{230^2}{d^2} = \frac{29}{1}$, or d = 42.6 mils. Ans.

Example 8.

A length of 1000 feet of wire 95 mils in

diameter has a resistance of 1.15 ohms what is the diameter of a wire of the same material of which the resistance of 1000 feet is 14 ohms.

Solution: This problem should be solved precisely as Example 7. $\frac{95^2}{d^2} = \frac{1}{1.15}$, or, d = 10.37 mils = the diameter of a wire 1000 feet long whose Resistance = 1 ohm. Then,

$\frac{\overline{10.37}^2}{d_1^2} = \frac{14}{1}$ and $d_1 = 27.7$ mils. Ans.

EXAMPLE 9.

Find the resistance in ohms of 1500 yards of copper wire 160 mils in diameter, the resistance of one mile of copper wire 230 mils in diameter being equal to one ohm.

Solution:

By Rule 4, the resistance of 1500 yards of copper wire, 230 mils in diameter = 1500 ÷ 1760 = .9036 ohms.

By Rule 5.

$\frac{.230^2}{.160^2} = \frac{R}{.9036}$ or R = 1.865 ohms. Ans.

Note.—Remember that there are 1760 yards in a mile.

Rule 6. The resistance of a conductor depends upon the physical properties of the material of which it is composed.

For instance, take two wires, one of copper, and one of iron, each one foot in length, and one mil

in diameter, the resistance of the copper wire is 9.6 ohms, and of the iron wire 58.45 ohms.

This resistance depending upon the material of the conductor, is called *Specific Resistance.*

It is expressed in various units as is indicated in table 1. The *Mil Foot* being the most convenient for general use.

Example 9.

The resistance of 3000 feet of German silver wire 200 mils in diameter is 9.44 ohms at O° C. Required, the *specific resistance* of German silver referred to the one mil foot unit at O° C.

Solution: Remember that the one mil foot unit means *the resistance of one foot of the material, having a circular cross section one mil in diameter.* Then from Rule 4, the resistance of one foot of the above wire is $\frac{9.44}{3000} = .0031466$ ohms. Then by Rule 5 the resistance of one foot, one mil in diameter will be $\frac{1_2}{200^2} = \frac{.003146}{R}$, where R = resistance of one mil foot. From this R = 125.89 ohms. Ans.

Note.—By referring to table 1, the *Specific Resistance* of German silver referred to the *one mil foot* unit will be found to be 125.89 ohms.

Note.—Division by the power of a number is indicated by using a *negative exponent.* Thus, 50×10^{-6} means $50 \div 10^{6}$.

Note.—A michrom is one millionth of an ohm.

Note.—The characters O^0 C, read zero degrees centigrade.

Example 10.

The resistance of a Hard Drawn copper rod of circular cross section 16′ long and 2″ in diameter was found to be 39.3 michroms at O° C. Required, the Specific Resistance of Hard Drawn copper at O° C referred to the centimetre, centimetre-unit, *i. e.* a bar of copper one centimetre long and one square centimetre in cross section. Equivalent values for the metric system are given in table 2.

Solution: Reducing the inches and feet to the metric system we have, $16 \times 30.479 = 487.664$ centimetres $2 \times 2 \times .7854 = 3.141$. $3.14 \times 6.451 = 20.262$ square centimetre $\frac{39.3}{487.66} = .0805$ Michroms. $\frac{.0805}{R} = \frac{1}{20.26}$, or, $R = 1.63$ Michroms, or, 1.63×10^{-6} ohms.

Note.—See table 1 of Specific Resistance.

Example 11.

Find the resistance at O° C of 15 miles of iron wire 300 mils in diameter.

Solution: From table 1, the specific resistance of iron referred to the *one mil foot* unit at O° C is found to be 58.45 ohms.

By Rules 4 and 5 we have
$15 \times 5280 \times 58.45 = 4{,}629{,}240$ ohms.

$\frac{300^2}{1^2} = \frac{4629240}{R}$ or $R = 51.43$ ohms. Ans.

EXAMPLE 12.

Find the resistance of 200 yards of hard drawn copper wire 134 mils in diameter.

Solution: From table 1, the Specific Resistance of hard drawn copper referred to the one mil foot unit is 9.829 ohms, then by rules 4 and 5 we have $200 \times 3 \times 9.829 = 5897.4$ ohms.

$$\frac{134^2}{1^2} = \frac{5897.4}{R} \text{ or } R = .328 \text{ ohms.} \quad \text{Ans.}$$

Resistance Referred to Weight.

It is often desirable to express the resistance of conductors of given length or cross section in terms of weight.

The weight of any body is equal to the product of its length by its cross section multiplied by a constant depending upon its specific gravity.

For instance the weight of a copper conductor is equal to its length in inches, multiplied by its cross section in inches $\times$ .32.

Note.— Copper weighs .32 lbs. per cu. inch.

Table 3 gives the weight per cubic inch for a variety of substances.

If we have two conductors of the same length, but different cross sections, it is obvious that the weight of the conductor will vary, *i. e.* the greater the cross section the greater the weight, and vice versa. From this we have the following:

Rule 7. The resistances of conductors of the same length vary inversely with their weights.

Example 13.

If one mile of a copper conductor weighing 100 pounds has a resistance of 10 *ohms*, what is the resistance of a conductor of the same length weighing 250 lbs.?

Solution: By Rule 7, since the length of the two conductors are the same the resistance will vary inversely as their weights, or,

$$\frac{10}{R} = \frac{250}{100} \text{ or } R = 25 \text{ ohms.} \quad \text{Ans.}$$

A portion of table 1 of Specific Resistance is devoted to the Foot-Grain unit, *i. e.* to giving the resistances of bodies of circular cross section one foot long and weighing one grain.

It is sometimes convenient to make use of this unit.

Example 14.

A certain annealed iron wire is 2000 feet long and weighs 200 lbs. Find its resistance.

Solution: From Table —, the Specific Resistance of annealed iron wire referred to the Foot-Grain unit is .1237 ohms, 2000 feet at this weight would weigh $2000 \times .1237 = 247.4$ ohms. By Rule 7 we have $\frac{247.4}{R} = \frac{2000 \times 700}{2000}$ $\therefore R = \frac{247.4}{700} = .3528$. Ans.

Conductance.

The passage of electric currents through bodies may be conceived to take place in either of two ways: —

First: It may be considered that owing to the physical properties of the body, *assistance* is given to the passage of the current, this is called conductance.

Second: It may also be considered that the passage of the current takes place in spite of opposition or resistance to its flow.

The latter conception, Resistance, is by far the more convenient in the majority of electrical problems, although Conductance is sometimes used in certain calculations.

Mathematically, conductance is the reciprocal of resistance.

Rule 8. To find the conductance of a body, the resistance being given, divide one by the resistance.

Example.

The resistance of a body is 4 ohms, what is the conductance?

Solution: By Rule 8, $1 \div 4 = .25$. Ans.

Variation of Resistance of Conductors Due to Change of Temperature.

The resistance of nearly all conductors increases with the rise and decreases with the fall of

temperature. A notable exception to this law is carbon.

The resistance of the materials given in Table 1 varies *with* the temperature and to calculate changes of resistance due to change of temperature we have the following rules.

Rule 9. To determine the increase of resistance of a conductor due to rise of temperature: Multiply the number of degrees centigrade rise, by the temperature co-efficient of the material; multiply the product by the original resistance and to this product add the original resistance.

Rule 10. To determine the decrease of resistance of a conductor due to fall of temperature: Multiply the number of degrees centigrade fall, by the temperature co-efficient of the material; multiply the product by the original resistance and subtract this from the original resistance.

EXAMPLE 15.

The resistance of a certain copper conductor at 12° C. was 250 ohms. Find the resistance of this conductor at 24° C.

Solution: By Rule 9 we have $250 + (250 \times 12 \times .00387) = 261.6$ ohms. Ans.

Note.—.00387 = co-efficient of temperature obtained from table 4.

EXAMPLE 16.

The resistance of a certain iron wire was found

to be 100 ohms at 50° C. Find the resistance of the same conductor at 0° C.

Solution: By Rule 10 we have $100 - (100 \times 50 \times .00511) = 74.45$ ohms. Ans.

Example 17.

The resistance of a certain copper wire is 24 ohms at 20° C. What is the resistance of this conductor at 60° C?

Solution: By Rule 9 we have $24 + (24 \times 40 \times .00387) = 27.715$ ohms. Ans.

Example 18.

The resistance of a certain German Silver wire is 150 ohms at 40° C. What is its resistance at 100° C?

Solution: By Rule 9, and referring to Table 4, we have $150 + (150 \times 60 \times .00044) = 153.96$ ohms. Ans.

By referring to Table 4, it will be noted that the variation of resistance per °C. for German Silver is much less than that of any other metal. It is for this reason that German Silver is generally used in rheostats and other apparatus when it is desired that the resistance shall be constant for ordinary variation in temperature.

Divided Circuits.

If a circuit be divided into two branches as at

A, in Fig. 1, uniting again at B, the *current* will also be divided, part flowing through one branch and part through the other.

The Joint Resistance of a Divided Circuit is the combined resistance of the branches into which the circuit is divided. Thus in Fig. 1 the Joint Resistance of the divided circuit from A to B is the combined resistance of C *and* D which is obviously less than the resistance of either C or D alone.

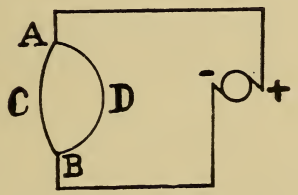

FIGURE 1.

Joint Conductance is the reciprocal of Joint Resistance.

The Joint Conductance of the divided circuit from A to B, Fig. 1, is the sum of the conductances of C and D. The reciprocal of this is the Joint Resistance.

Rule 11. To find the Joint Resistance of a Divided Circuit, find the Joint Conductance of the different branches of the circuit. The reciprocal of this will be the Joint Resistance.

Example 19.

Find the Joint Resistance of 2 wires of 2 and 4 ohms' resistance, respectively.

Solution: By Rule 8. Conductance of the wires $= \frac{1}{2}$ and $\frac{1}{4}$, respectively.

By Rule 11. Joint Conductance $= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.
Joint Resistance $= 1 \div \frac{3}{4} = 1.33$ ohms. Ans.

Example 20.

Find the Joint Resistance of three wires whose resistances are 1.3, 2.7 and 4 ohms, respectively.

Solution: Joint Conductance $= \frac{1}{1.3} + \frac{1}{2.7} + \frac{1}{4} = .77 + .372 + .25 = 1.392$. $\therefore$ Joint Resistance $= 1 \div 1.392 = .718$ ohm. Ans.

Example 21.

The Joint Resistance of two conductors A and B is 2 ohms; the resistance of A is 7 ohms, find the resistance of B.

Solution:

Joint Resistance $= 2$ ohms.
$\therefore$ Joint Conductance $= 1 \div 2 = \frac{1}{2}$
Conductance of A $= 1 \div 7 = \frac{1}{7}$
$\therefore$ Conductance of B $= \frac{1}{2} - \frac{1}{7} = \frac{5}{14}$
$\therefore$ Resistance of B $= 1 \div \frac{5}{14} = 2\frac{4}{5}$ ohms. Ans.

Strength of Current in Branches of Divided Circuits.

Referring again to Fig. 1, if C and D be con-

ductors of like resistance, it is obvious that the current in each will be the same.

If, however, the resistance of C is greater than that of D, the current in C will be less than the current in D. To express this in a convenient form we have the following:

Rule 12. The relative strength of current in the different branches of a divided circuit is directly proportional to the conductivity of the branches, or inversely proportional to their resistance.

Example 22.

Three wires offering resistances of 6, 7 and 15 ohms, respectively, are joined in parallel. Find the proportion of current which would flow in each branch.

Solution: By Rules 11 and 12, Joint Conductance $= \frac{1}{6} + \frac{1}{7} + \frac{1}{15} = \frac{1}{6} + \frac{1}{7} + \frac{1}{15} = \frac{35+30+14}{210} = \frac{79}{210}$.

Supposing from this the *current* to divide in 79 parts: 35 of these parts or $\frac{35}{79}$ will go through the first wire, $\frac{30}{79}$ through the second wire, and $\frac{14}{79}$ through the third wire. Ans.

Example 23.

A current of 42 amperes flows through three conductors in *parallel* of 5, 10 and 20 ohms' resistance respectively. Find the current in each conductor.

Solution: Joint Conductance $= \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = \frac{4+2+1}{20} = \frac{7}{20}$.

Supposing the current to be divided into 7 parts, 4 of these parts would flow in the first conductor 2 in the second and 1 in the third.

The whole current is 42 amperes.

$\frac{4}{7}$ of $42 = 24$.
$\frac{2}{7}$ of $42 = 12$.
$\frac{1}{7}$ of $42 = 6$.

Current in first Conductor $= 24$ amperes.
" " second " $= 12$ "
" " third " $= 6$ "
} Ans.

CHAPTER III.

ENERGY OF ELECTRIC CURRENTS.

FORCE.

Force is that which changes, or tends to change the state of a body with reference to rest or motion: as for instance — gravity, friction, cohesion. The mechanical unit of force, is the pound Avoirdupois. *The unit of force used in connection with electrical problems is the Dyne. It is that force which will give to a mass of one gramme in one second a velocity of one centimetre per second.*

WORK.

Work, mechanically, is overcoming resistance. It requires a certain amount of work to move a force of one pound one foot and twice that amount of work to move a force of two pounds one foot, or one pound two feet.

The mechanical unit of work is the *foot-pound* and represents the amount of work required to to move a force of one pound one foot.

The unit of work used in electrical problems is the *Erg.* It is the amount of work required to move a force of one dyne one centimetre.

One foot-pound equals about thirteen and one-half million ergs.

ENERGY.

Energy is a term for expressing the ability of an agent to do work. The units of energy are the same as the units of work, for the energy of a body is measured by the work it can do.

POWER.

Power is the *rate* of doing work. The most common unit of power is the *Horse-power*, which is equal to *33,000 foot-pounds per minute*; that is, a a machine having a capacity of one horse-power is able to do the work of lifting 33,000 pounds one foot in one minute, or its equivalent.

The unit of power used in all electrical problems, is the *Watt*. It is the rate of doing work equal to *10,000,000 ergs per second*, and is also equal to .00134 horse-power, or one horse-power is equal to 746 Watts.

HEAT.

The theory of heat rests upon the supposition that it consists of, or is the result of, the motion of the molecules of a body; and that to produce this motion requires the expenditure of a definite amount of mechanical energy. This supposition is confirmed by the experiments of Joule, who produced heat by friction between rigid surfaces; by the compression of air and by the agitation of water.

The energy represented by 772 foot-pounds or $\frac{1}{4\,2\,8}$ of a horse power, if it were all converted into

heat would raise the temperature of one pound of water one degree Fahrenheit. This number is called "Joule's Equivalent."

The unit of heat used almost entirely in electrical work is the *Calorie.* This is determined by experiment as in the case of the unit previously given. It is the heat required to raise one gramme of water from 0° centigrade to 1° centigrade.

1 Calorie = 41,595,000 ergs: that is, it requires an expenditure of 41,595,000 ergs of work to raise one gramme of water from 0° centigrade to 1° centigrade. The calorie is also called "*Joule's equivalent for C. G. S. system of units.*"

The work, (W.) done by a current of electricity in moving C. units of electricity over a conductor at an electromotive force (E.) is

$$W. = C. E. t.$$

Where C. equals the number of amperes, E. equals the E. M. F. and t, is time in seconds during which the current acts.

Therefore the work of an ampere for one second at the pressure of one volt is

$$W. = 100{,}000{,}000 \times .1 \text{ Ergs} = 10{,}000{,}000 \text{ Ergs.}$$

But from the definition on page 30, the Watt equals 10,000,000 Ergs per second: Therefore a Watt is equal to a Volt-Ampere or Watts = C. E.

By Rule 2. E. = C. R. Substituting this value of E, in the equation

$$\text{Watts} = \text{C. E.}$$
$$\text{Watts} = \text{C}^2 \text{ R.}$$

Rule 13. The number of ergs expended in forcing a current of electricity over a conductor is equal to the product of the number of amperes of current flowing in the conductor, by the electromotive force in volts multiplied by the time during which the current flows, divided by 10,000,000, or letting

W. = The work.
C. = The current in amperes.
E. = The electromotive force in volts.
t. = The time in seconds, and we have
$W. = E. C. t.$

EXAMPLE 1.

A certain conductor has a current of 20 amperes flowing through it at an electromotive force of 110 volts. Required the amount of work expended in three seconds.

Solution: By Rule 13; and remembering that 1 volt = 100,000,000 c. g. s. unit and that 1 ampere = .1 of an c. g. s. unit we have by substitution

$W. = 20 \times .1 \times 100{,}000{,}000 \times 110 \times 3 =$ 66,000,000,000 ergs. Ans.

EXAMPLE 2.

Express the amount of work done in the foregoing problem in foot-pounds.

Solution: From table No. 2, we find that one foot-pound = 13,562,000 ergs: Therefore, Work = $\frac{66,000,000,000}{13,562,000}$ = 4,860 foot-pounds. Ans.

Rule 14. The amount of power in Watts required to force a current of electricity over a conductor is equal to the product of the current in amperes by the electromotive force in volts, or letting W.= the power in watts, C. = the current in amperes and E.= the electromotive force in volts; we have W.= E. C.

Example 3.

A certain generator delivers a current of a thousand amperes at an electromotive force of 500 volts. Required the watts delivered.

Solution: By Rule 14, W = 1,000 × 500 = 500,000 Watts. Ans.

Example 4.

One thousand incandescent lamps are running in parallel on a certain circuit: Each lamp requires one-half an ampere at an electromotive force of 110 volts. Find the total watts required by these lamps.

Solution: By Rule 14. Since every lamp requires one-half an ampere, the total number of amperes required by one thousand lamps would be 1,000 × .5, = 500 amperes.

$\therefore$ Watts $= 500 \times 110 = 55{,}000$ Watts.

Note.—A Kilowatt is equal to one thousand watts and this unit is frequently used as it is more convenient in many cases than the watt.

Rules 15 and 16, which follow are produced directly by combining Rule 14, with Rules 1 and 2.

Rule 15. The power expended in forcing a current of electricity through a conductor is equal to the square of the current by the resistance of the conductor.

That is, $Watts = C^2 R$.

EXAMPLE 5.

A current of 9 amperes is sent through a conductor, whose resistance is 7 ohms. Required the power in watts.

Solution: Power $= C^2 R = 9 \times 9 \times 7 = 561$ Watts. Ans.

Rule 16. The amount of power required to force a current of electricity through a conductor is equal to the square of electromotive force in volts divided by the resistance in ohms. i. e. $W = \frac{E^2}{R}$

EXAMPLE 6.

A conductor whose resistance is .08 of an ohm is carrying a current at an electromotive force of 80 volts. Required the power to maintain the current at this resistance.

Solution: Power $= \frac{E^2}{R} = \frac{80 \times 80}{.08} = 80{,}000$

Example 7.

The resistance of the external circuit of a dynamo is .8 of an ohm: The current flowing through the conductor is 800 amperes. What power is expended in maintaining this current at the given resistance?

Solution: Power $= C^2 R = 800 \times 800 \times .8 =$ 512,000 Watts or 512 Kilowatts. Ans.

Efficiency.

The *commercial efficiency* of an electrical appliance whether *dynamo*, *motor*, *transformer or conductor* is the ratio of the energy delivered by the appliance to the energy delivered to it.

The *electrical efficiency* of an appliance is the ratio of the amount of electrical energy delivered to the *external circuit* to the amount of electrical energy delivered to the appliance.

Heat Lost in a Conductor.

An expression of the work done in forcing a current over a conductor is given on page 31 thus: $W = C^2 Rt$. It has been determined that the number of heat units developed in a conductor is proportional.

1. To its resistance.

2. To the square of the strength of the current.
3. To the time that the current lasts.

It should be noted that the calorie, a definition of which is given on page 31, is the same as the *gramme calorie* or "Joule," that is: the amount of heat required to raise the temperature of one gramme of water 1^0 centigrade is sometimes termed "a water-gramme degree centigrade."

Rule 17. The amount of heat developed in a conductor by a current of electricity is equal to the watts multiplied by .24, or letting ***H*** *equal the number of heat units developed, t, the time during which the current flows and C, R, E, equal their usual values, we have the three expressions:*

1. $H = C^2 Rt \times .24.$
2. $H = C\ Et \times .24.$
3. $H = \frac{E^2 t}{R} \times .24.$

EXAMPLE 8.

The electromotive force at the ends of the two carbons of a certain arc lamp is 48 volts. A current of 8 amperes is passing through it. Required the amount of heat developed per second.

Solution: By Rule 17 formula 2, we have $H = E\ C \times .24$, substituting in the expression the quantities given in the above example, we have $H = 48 \times 8 \times .24 = 92.16$ heat units or calories. Ans.

EXAMPLE 9.

A certain incandescent lamp has a resistance of 220 ohms. The electromotive force at the terminals of the lamp is 112 volts: Required the amount of heat developed in the lamp per second.

Solution: By Rule 17, formula 3, we have $H = \frac{E^2}{R} \times .24 = \frac{112 \times 112}{220} \times .24 = 13.68$ calories. Ans.

CHAPTER IV.

Batteries.

The commercial value of a battery is determined by the cost of the material consumed per unit of electrical energy delivered to the external circuit. The electromotive force of a given cell depends merely upon the nature of the metals of the battery, not upon its size or the distance between the plates. The same electromotive force, however, does not always produce the same current, as the current depends not only upon the electromotive force tending to drive it around the circuit, but also upon the resistance which it has to encounter and overcome in its flow.

The amount of chemical action in the cell is proportional to the quantity of electricity that passes through it, that is: proportional to the number of amperes multiplied by the time during which the current flows. Thus, one coulomb (that is one ampere for one second) in passing through a cell liberates $\frac{1}{96,600}$ gram of hydrogen. The corresponding weights of other elements or compounds liberated or deposited are found by multiplying the weight of hydrogen liberated, by its chemical equivalent and dividing by the valency of element

or metal of the base of the compound in question. Thus in a cell having zinc for the oxidant we would multiply $\frac{1}{96,600}$ of a gram by the atomic weight of zinc, which is 65, divided by the valency which is two. This would show that $\frac{32.5}{96,600}$ grams of zinc are dissolved by the passage of one coulomb of electricity. Corresponding weights of other materials consumed in batteries can be calculated by referring to the table of electro-chemical-equivalents No. 5.

It has been stated that the electromotive force of a cell depends upon the material of its plates. It was shown in the preceding chapter that the energy in an electrical circuit is equal to the current multiplied by the electromotive force. We have also seen that the current of a battery is proportional to the amount of chemical action of the elements of the battery. The electromotive force of the current is proportional to the thermo-electric-equivalent of these elements, that is: to "the energy of combination" of these elements. In other words, whenever chemical compounds are decomposed or formed a certain definite amount of heat is developed, always the same for the same weight and combination. This heat represents the energy of the combination; and also represents a corresponding amount of electric energy which is proportional to the product of the current and the electromotive force. The electromotive force can therefore be

determined by expressing the electric energy in watts and dividing by the amount of current corresponding to the amount of metal dissolved.

The electro-chemical-equivalents are proportional to the chemical equivalents of the substance, that is: to the relative weights of the substances which take part in chemical reaction. If, therefore, we know the electro-chemical-equivalent of hydrogen, the other electro-chemical-equivalents may be found by multiplying this by their chemical equivalents.

Rule 18. The electro-chemical-equivalent of an element is found by multiplying the electro-chemical-equivalent of hydrogen by the chemical-equivalent of the element.

Example 1.

The chemical equivalent of copper (Cu) is 31.5: find its electro-chemical equivalent in grams per coulomb.

Solution:

By referring to Table No. 5, we find the electro-chemical-equivalent of hydrogen to be .000,010,352.

Then by Rule 18, we have, $31.5 \times .000,010,352 = .000,326,1$. Ans.

Example 2.

The chemical-equivalent of oxygen is 8: find the

electro-chemical-equivalent in grams per coulomb.

Solution as above: $8 \times .000,010,352 = .000,082,8$. Ans.

The electro-chemical-equivalent of hydrogen in grams per coulumb is .000,010,352. If now the heat of combination of one chemical-equivalent of an ion is H, then .000,010,36 = zh, consequently

$$E = 4.16 \times .000,010,352, \text{ zh.}$$

$$\text{Or, } E = .000,043, \text{ Z H.}$$

It is only necessary then to find the algebraic sum of the heats of combination for a chemical-equivalent of each ion taking part in the re-action, in order to find the electromotive force in volts.

Note.—Ions are groups of atoms which result from the electrolytic decomposition of a molecule. They are electro-positive and electro-negative. The electro-positive ion appears at the plate connected with the electro-negative terminal or Kathode, and is called the Kathion. The electro-negative ion appears at the plate connected with the electro-positive terminal or Anode and is called the Anion.

If, in the formula, electromotive force becomes unity, then the number of calories corresponding to one volt is the reciprocal of the constant .000,043 or 23,200. In this discussion the chemical-equivalents used are half atomic weights of bivalent substances corresponding to one of Hydrogen which is univalent. With this condition one volt is equal to 23,200 calories. If the chemical-equivalents used are the atomic weight of bivalent ele-

ments and double those of the univalent ones, then a volt is memerically equal to 46,400 calories.

Rule 19. To find the electromotive force developed by the combination of chemicals as necessary for the disintegration of a chemical compound, find the difference between the number of heat units of disintegration and formation, multiply this by .000,043.

Note.—In the following examples the thermal values are taken from Thomson's determinations.

Example 3.

Find the electromotive force of the Smee Cell.

Solution:

In this cell the chemical action consists in the formation of zinc sulphate and hydrogen from zinc and sulphuric acid as is shown by the reaction.

$$Zn + H_2 S O_4 = Zn S O_4 + H_2$$

Heat of formation of
$Zn O_2$ and $S O_2$ into $Zn S O_4 = 79{,}495$ calories.

Heat of disintegration of
$H_2 O_2$ and $S O_2$ from $H_2 S O_4 = 60{,}920$ calories.

Applying Rule 19, we have,
$(79{,}495 - 60{,}920) \times .000043 = .80$ volts. Ans.

Example 4.

Find the electromotive force of the Daniell Cell.

Solution:

In this cell zinc sulphate is formed and copper sulphate disintegrated, the re-action is:

$$Zn + Zn\,S\,O_4 + Cu\,S\,O_4 = 2\,Zn\,S\,O_4 + Cu$$

Heat of formation of
Zn, O_2 and S O_2 into Zn S O_4 =79,495 calories.

Heat of disintegration of
Cu, O_2 and S O_2 from Cu S O_4 = 55,745 calories.

Applying Rule 19, we have,
(79,495 — 55,745) × .000043 = 1.02 volts. Ans.

Example 5.

Find the electromotive force of the Bunsen Cell.

In this cell, sulphuric($H_2\,S\,O_4$) and nitric($H\,N\,O_3$) acids are disintegrated and zinc sulphate ($Zn\,S\,O_4$) water ($H_2\,O$) and peroxide of nitrogen ($N\,O_2$) are formed. The chemical reaction is as follows:

$$Zn + H_2\,S\,O_4 + 2\,H\,N\,O_3 = Zn\,S\,O_4 + 2\,H_2\,O + 2\,N\,O_2$$

Heat of formation of
Zn, O_2 and S O_2 into Zn S O_4=79,495 calories.

Heat of formation of
2 (H_2 O) into 2 H_2 O =68,369 calories.

Heat of formation of
2 (N O,) into 2 N O_2 =19,570 calories.

Total heat of formation 167,425 calories.

Heat of disintegration of
H_2, O_2 and S O_2 from H_2 S O_4 = 60,920 calories.

Heat of disintegration of
2 (H, N O, O_2 from 2 H N O_3 = 63,185 calories.

Total heat of disintegration 124,105 calories.

Applying Rule 19, we have,
(167,425—124,105) × .000043 = 1.863 volts. Ans.

EXAMPLE 6.

Find the electromotive force of the Silver Chloride Cell.

Solution:

Assume the cell set up with a dilute solution of zinc sulphate. The result of the action taking place when the cell is in operation is the formation of zinc chloride and the decomposition of silver chloride; hence we have only to find the difference between the heat of formation of the chlorides.

Heat of formation of Zn and Cl_2=56,429 calories.

Heat of disintegration of Ag_2 Cl_2=29,380 calories.

By Rule 19, we have,
(56,420—29,380) × .000043 = 1.16 volts. Ans.

Rule 20. To find the weight of an element consumed in a cell per coulomb of electricity: Multiply the electro-chemical-equivalent of the element by the

number of atoms of the element enteriny into the reaction.

$$G = K\,N$$

Note—In the foregoing expression G represents the weight, K the electro-chemical-equivalent and N the number of atoms of the element.

Rule 21. To find the weight of an element consumed in a cell per Watt-Hour: Apply Rule 20; divide the result by the E. M. F. of the cell and multiply this result by 3,600 to obtain weight in grams, or by 7.92 to obtain weight in pounds.

Weight in grams, (1) $G = \frac{K\,N}{E} \times 3{,}600$

Weight in pounds, (2) $P = \frac{K\,N}{E} \times 7.92$

Note.—In the preceding rule the electro-chemical-equivalent should be taken in grams per coulomb.

Note.—A Watt-Hour is a Watt for an hour.

EXAMPLE 7.

Find the weight in grams of zinc consumed in a Smee cell per watt-hour, in grams and in pounds.

Solution: The electro-chemical-equivalent of zinc is found from Table 5 to be .000364 and one atom of the element is found by inspecting the reaction into it.

$$\therefore G = \frac{K\,N}{E} \times 3{,}600 = \frac{.000364 \times 1 \times 3{,}600}{.8} = 1.6 \text{ grams. Ans. 1.}$$

or $P = \frac{K\,N}{E} \times 7.92 = \frac{.000364, \times 1 \times 7.92}{.8} = .0032$ lbs. Ans. 2.

EXAMPLE 8.

Express the last answer in pounds per horse power.

Solution: One-horse power = 746 watts.

$\therefore .0032 \times 746 = 2.38$ lbs. Ans.

EXAMPLE 9.

Find the weight of sulphuric acid ($H_2\,S\,O_4$) consumed in the Smee cell in pounds per watt-hour.

Solution: The sulphuric acid should be treated as an element with respect to Rule 21.

From Table 5, K, for Hs SO_4 equals .000507.

$\therefore P = \frac{K\,N}{E} \times 7.92 = \frac{.000507 \times 1}{.8} \times 7.92 = .005$. Ans.

ELECTROLYSIS.

Electrolysis is the process of decomposing a liquid by means of a current of electricity.

It is the reverse of the action in the Voltaic cell where chemicals are consumed to produce or generate a current.

Rule 22. To find the weight in grams of an element deposited by a current of electricity: Multiply the electro-chemical-equivalent of the element, by the current in amperes, and by the time in seconds, during which the current flows.

$$G = K\,C\,t$$

Example 10.

A current of .5 amperes was passed through an electrolytic cell containing a solution of silver for 10 minutes; find the weight of silver deposited in that time.

Solution: $G = K\ C\ t$ or

$G = .0011180 \times .5 \times 10 \times 60 = .3354$ grams. Ans.

Example 11.

A current of 3 amperes was passed through an electrolytic cell containing acidulated water for 60 minutes.

Find the weight of hydrogen evolved.

Solution: $G = K\ C\ t$ or

$G = .000010352 \times 3 \times 60 \times 60 = .111$ grams. Ans.

Grouping of Cells.

The best grouping of a given number of cells to accomplish a certain result, depends upon whether it is to be obtained with maximum activity or maximum economy.

Maximum activity involves the most rapid conversion of the energy applied into the energy of an electric current, that is: that the work shall be performed with the greatest celerity.

Maximum economy requires conditions so arranged, that the work may be performed with the least lost, that is: that it shall be done with the greatest economy.

In the first place economy is sacrificed to time; in the second, time is sacrificed to economy.

Note.—The internal resistance of a cell is that resistance in the cell between its electrodes.

Application of Ohm's law to a single cell.

Rule 23. The current from a single cell is equal to the electromotive force of the cell, divided by (the resistance of the external circuit plus the internal resistance of the cell.)

$$C = \frac{E}{R + r}$$

Note.—In the foregoing expression, C equals the current in amperes, E the electromotive force in volts, R the external resistance of the circuit and r the internal resistance of a cell.

EXAMPLE 12.

The electromotive force of a cell is one volt; its internal resistance is 2.8 ohms; find the strength of current when the external resistance is 5 ohms.

Solution:

$$C = \frac{E}{R + r} = \frac{1}{2.8+5} = \frac{1}{7.8} = 128 \text{ amperes. Ans.}$$

EXAMPLE 13.

A cell whose electromotive force is 1.6 volts and whose internal resistance is .3 ohm, is joined up in simple circuit with a galvonometer whose resistance is 120 ohms, and has a standard resistance of 15 ohms.

Find the strength of current in the circuit.

Solution:

$$C = \frac{E}{R} \frac{1.6}{.3\times120\times15} = \frac{1.6}{135.3} = .011 \text{ amperes.} \quad \text{Ans.}$$

Example 14.

There are 15 cells each having an electromotive force of 15 volts and an internal resistance of .2 ohms. What would be the strength of the current if 3 of the cells were joined up in opposition to the others?

Solution:

In this case the electromotive force of the battery is equal to that due to the 12 cells joined together minus that of the 3 cells joined together, but in opposition to the 12. Therefore,

$$C = \frac{E}{R+r} = \frac{(1.5\times12)-(1.5\times3)}{.2 \times 15} = \frac{13.5}{3} = 4.5 \text{ amperes.} \quad \text{Ans.}$$

CELLS IN SERIES.

A battery of cells may be grouped in several ways. When connected in series, the positive terminal of one cell is joined to the negative of the next and its negative to the positive of the preceding one and so on. Thus arranged the total electromotive force of the battery, is the sum of the electromotive forces of the similar cells and the entire internal resistance is correspondingly greater since the current must pass in through all the cells connected in the series.

Rule 24. The current of a battery having a number of cells arranged in series is equal to the total electromotive force of the battery divided by (the total internal resistance of the battery plus the resistance of the external circuit.)

$$C = \frac{n\,E}{n\,r + R}$$

where n = number of cells in series.

EXAMPLE 15.

The electromotive force of a cell is 1 volt. The internal resistance is 2.8 ohms. Find the strength of current of 6 of these cells joined in series, the external resistance being 5 ohms.

Solution:

$$C = \frac{n\,e}{n\,r + R} = \frac{1 \times 6}{(2.8 \times 6) + 5} = \frac{6}{21.8} = .27 \text{ amperes.} \quad \text{Ans.}$$

EXAMPLE 16.

The electromotive force of a cell is 1.5 volts, and its internal resistance is 2 ohms. Find the strength of current when 15 of these cells are arranged in series, the external resistance of the circuit being 20 ohms.

Solution:

$$C = \frac{n\,E}{n\,r + R} = \frac{15 \times 1.5}{(15 \times .2) + 20} = .978 \text{ amperes.} \quad \text{Ans.}$$

CELLS IN PARALLEL.

The effect of this grouping is the reduction of the resulting internal resistance directly in proportion to the number of cells. The positive terminals of the cells are all joined together; likewise the negative terminals of all the cells.

The two terminals formed by this joining constitute the main terminals of the battery.

The effect is precisely as though one large cell were replaced by a number of smaller cells, the sum of the areas of whose plates would be equal to the area of the plates of the single cell.

Rule 25. The current of a battery whose cells are arranged in parallel, is equal to the electromotive force of a single cell divided by the external resistance of the circuit plus the internal resistance of a cell divided by the number of cells.

$$C = \frac{E}{\frac{r}{n} + R}$$

Note.—Here, n equals the number of cells in parallel.

EXAMPLE 17.

Find the strength of current when 6 cells are joined in parallel; the electromotive force of each cell being 1 volt, the internal resistance 2.8 ohms and the external resistance .5 ohms.

Solution:

$$C = \frac{E}{\frac{r}{n} + R} = \frac{1}{\frac{2.8}{6} + .5} = \frac{1}{.96} = 1.04 \text{ amperes.} \quad \text{Ans.}$$

Example 18.

The electromotive force of a cell is 1 volt; its internal resistance is 30 ohms. Find the strength of current from 20 of these cells connected in parallel, the external resistance being 1.5 ohms.

Solution:

$$C = \frac{E}{\frac{r}{n} + R} = \frac{1}{\frac{30}{20} + 1.5} = \frac{1}{3} = .33 \text{ amperes.} \quad \text{Ans.}$$

Cells in multiple series.

A battery of n cells may be arranged in series, l cells in each series and m cells in parallel; then $n = m\ l$, that is: the total number of cells in the battery equals the product of the number in series multiplied by the number in parallel. The positive terminals of the several series should be coupled together, also the negative terminals. The electromotive force of the battery is increased in proportion to the number of cells in series, that is: it is l times as great as that of a single cell and the resistance of each series is increased in the same proportion. The internal resistance however, owing to the increased number of paths, is reduced directly in proportion to the number of series.

Rule 26. The current of a battery whose cells are arranged in multiple series is equal to the electromotive force of the cell multiplied by the number of cells in each series divided by (the internal resistance of a cell multiplied by the number of cells in series divided by the numbers of series plus the external resistance of the circuit).

$$C = \frac{l\,E}{\frac{l\,r}{m} + R}$$

Note.—Remember that in the foregoing formula—m = the number of cells in parallel or number of series; l = the number of cells in each series.

EXAMPLE 19.

Find the strength of the current of a battery of 6 cells arranged 3 in series and 2 in parallel. The electromotive force of a cell is 1.3 volts and the internal resistance 5 ohms.

Solution:

$$C = \frac{l\,E}{\frac{l\,r}{m} + R} = \frac{3 \times 1.3}{\frac{3 \times .5}{2} + 0} = 5.2 \text{ amperes.} \quad \text{Ans.}$$

EXAMPLE 20.

A battery of 32 cells is arranged 16 in series and 2 in parallel. The resistance of the external circuit is 12 ohms, the electromotive force of a cell 1 volt and its internal resistance 1.5 ohms. Find the current in the circuit.

Solution:

$$C = \frac{l\,E}{\frac{l\,r}{m} + R} = \frac{16 \times 1}{\frac{16 \times 1.5}{2} + 12} = \frac{16}{24} = .66 \text{ amperes.} \quad \text{Ans.}$$

BATTERY, POWER AND BEST ARRANGEMENT OF CELLS.

For steady current of maximum value the internal resistance of the battery should equal the external resistance of the circuit. This is also the condition for maximum activity for a fixed external resistance. The efficiency will be about 50 %, since one-half the total energy is wasted internally and one-half externally if none is stored up by electrolysis or by the motor mechanism.

Rules that have been made to be used in calculating the number and best arrangement of cells to obtain a certain current in a battery, have proved unsatisfactory. The writer therefore in consideration of this has thought it best to cite a number of examples, explaining each one in full.

Example 21.

If the electromotive force of a cell is 3 volts and its internal resistance 1 ohm, how many of these cells in series would be required to produce a current of 2.5 amperes through a circuit which has a resistance of 14.3 ohms.

Let x denote the number of cells required, then $3x$ will be the electromotive force of the battery, and $1x$ its internal resistance.

$$C = \frac{E}{R}$$

$$2.5 = \frac{3x}{x + 14.3}$$

$$3x = 2.5\,x + 35.75$$

$$5x = 35.75$$

$$x = 71.5$$

Ans. 72 cells.

EXAMPLE 22.

How many cells arranged in series, each having an electromotive force of 1.5 volts and internal resistance .2 ohms, are necessary to produce 2 amperes of current in a circuit offering 7.8 ohms' resistance?

Let x denote the number of cells required.

$$C = \frac{E}{R}$$

$$2 = \frac{1.5\,x}{.2\,x + 7.8}$$

$$1.5\,x = .4\,x + 15.6$$

$$1.1\,x = 15.6$$

$$x = \frac{15.6}{1.1} = 14.18 = 15 \text{ cells. Ans.}$$

EXAMPLE 23.

How many cells, arranged for quantity, would be required to produce a current of 1 ampere through an external resistance of 1 ohm, the electromotive force of each cell being 2 volts, and its internal resistance 5 ohms?

By joining cells in parallel, the resistance in the battery itself is reduced, but no additional power is given to overcome the resistance in any other part of the circuit. In other words, you only get the electromotive force of 1 cell, but you reduce the resistance of the battery by the number you join in parallel.

Let x denote the number of cells required.

$$C = \frac{E}{R}$$

$$1 = \frac{2}{\frac{5}{x}+1}$$

$$2 = \frac{5}{x}+1$$

$$2x = 5 + x$$

$$x = 5. \quad \text{Ans. } 5 \text{ cells.}$$

EXAMPLE 24.

How many cells, arranged for quantity, each having an electromotive force of 1 volt, and internal resistance 30 ohms, will produce $\frac{1}{3}$ ampere of current through an external resistance of 2 ohms?

Let x denote the number required.

$$C = \frac{E}{R}$$

$$\frac{1}{3} = \frac{1}{\frac{30}{x}+2}$$

$$3 = \frac{30}{x} + 2$$

$$3x = 30 + 2x$$

$$x = 30. \text{ Ans. } 30 \text{ cells.}$$

EXAMPLE 25.

If the internal resistance of a cell is 1 ohm and you have 90 such cells, how would you arrange them so as to send the strongest current through an external resistance of 10 ohms?

The most advantageous method of combining cells is to arrange them so that the internal resistance of the battery is equal to the external resistance of the circuit.

Let x denote the number of cells in series, and y the number in multiple.

Then $x + y$, or the number of cells in the battery, will equal 90; and $\frac{x \times 1}{y}$ the internal resistance of the battery, ought to be equal to 10 ohms to get the best effect.

$$\text{Let } \frac{x \times 1}{y} = 10$$

$$\therefore \quad x = 10y \quad . \quad . \quad (1)$$

$$\text{But } x \times y = 90 \quad . \quad . \quad (2)$$

Substituting 10 y (the value of x) for x in equation (2).

$$10\,y \times y = 90$$
$$y^2 = 9$$
$$y = 3$$
But $$x = 10y$$
$$x = 30$$

Ans. 30 in series and 3 in multiple.

Example 26.

If the electromotive force of a cell is 2 volts, and its internal resistance 4 ohms, how many cells will be required to produce 1 ampere of current through an external resistance of 15 ohms?

Let x denote the number in series, and y the number in multiple.

$$\text{Since } C = \frac{E}{R}$$
$$1 = \frac{\frac{x \times 2}{y}}{\frac{x \times 4}{y} + 15}. \qquad (1)$$

To get the best effect $\frac{x \times 4}{y}$ must equal 15 (2)

$$\therefore 4x = 15y$$
$$x = \frac{15\,y}{4}$$

Substituting this value for x in equation (1)

$$\frac{\frac{\frac{15\,y}{4} \times 2}{y}}{\frac{\frac{15\,y}{4} \times 4}{y} + 15} = 1$$
$$\frac{\frac{15\,y}{2}}{\frac{15\,y}{30}} = 1$$

$$15y = 60$$
$$y = 4$$
$$\text{But } x = \frac{15\,y}{4}$$
$$\therefore x = 15$$

Ans. 60 cells arranged 15 in series and 4 abreast.

EFFICIENCY OF A BATTERY.

Rule 27. To find the efficiency of a battery, divide the resistance of the external circuit by the resistance of the external circuit plus the internal resistance of the battery and multiply by 100.

Letting Eff = Efficiency, we have

$$\text{Eff.} = \frac{R}{R + r} \times 100$$

EXAMPLE 27.

What is the efficiency of a battery of 50 cells, the external resistance of the circuit being 20 ohms, the internal resistance of each cell being .2 ohms.

Solution: Applying the foregoing rule we have:

$$\text{Eff} = \frac{R}{R + r} \times 100 = \frac{20 \times 100}{20 + (50 \times .2)} = \frac{20 \times 100}{30} = 66\tfrac{2}{3}\%.$$

Ans.

CHAPTER V.

THE MAGNETIC CIRCUIT.

The magnetic field is the space surrounding a magnet, in which its magnetic force can be detected.

LINES OF FORCE.

The strength of the magnetic force at any point in the magnetic field is measured by the number of lines of force per square inch or per square centimeter at that point, and the direction in which the force acts is shown by the direction of the lines of force.

A plane is said to be pierced by one line of force per square centimeter when a unit magnetic pole placed in that plane is acted on with a force of one dyne.

The direction of the magnetic force is that in which it will drive the north pole of a magnet, and the lines of force are said to run in the direction of the magnetic force.

Lines of force are always closed curves. They emerge from the north pole of a magnet, curve around the space outside, and entering the south pole, pass through the substance of the magnet to the north pole, thus completing the circuit. This

complete curve formed by the lines of force is called the magnetic circuit.

MAGNETISM IN AIR.

A coil of wire in which a current of electricity is flowing, sets up a field of force in the space which it encloses. The lines of force, passing through this space, emerge from one end of the coil, and passing around the outside of it, enter the other end, thus completing the magnetic circuit. The number of lines of force which will be set up in the enclosed space, depends on the area of the space, on the number of turns per inch in length of the coil and on the amount of current flowing in it.

Rule 28. The number of lines of force per square inch that will be set up in a space containing no metal and enclosed by a coil of wire carrying a current of electricity, is equal to the product of the number of turns per inch in length of the coil, multiplied by the number of amperes flowing through it, multiplied by 3.2 (or more exactly 3.19183.)

Example 1.

A helix, 36 inches long, consisting of 288 turns has a current of 3 amperes flowing through it. How many lines of force per square inch are there inside the coil?

Solution:

There are $\frac{28.8}{3.6} = 8$ turns per inch in length of the coil. Therefore, there are $8 \times 3 \times 3.2 = 76.8$ lines of force per square inch in the coil.

The number of lines of force per square inch or per square centimeter is called induction per square inch or per square centimeter.

DEFINITION OF "AMPERE-TURN."

Since the number of lines of force set up by a current flowing through a helix varies as the number of turns in the helix, and as the number of amperes flowing through it, the expression "ampere-turns" has been taken as a unit.

Thus, the number of ampere-turns per inch in length (i. e. the number of amperes multiplied by the number of turns per inch in length) multiplied by 3.2 equals the number of lines per square inch in the coil.

EXAMPLE 2.

Suppose we wish to obtain an induction of 500 lines per square inch with a current of 10 amperes. How many turns to the inch must we have?

Solution:

Since each ampere-turn produces 3.2 lines of force, we must have $\frac{500}{3.2} = 156.2$ ampere-turns. Then since we have 10 amperes, $\frac{156.2}{10} = 15.62$ is the number of turns per inch required.

Example 3.

How many amperes will be required to produce 1,600 lines per square inch in a helix 14 inches long consisting of 2,600 turns?

Solution:

We will require $\frac{1600}{3.2} = 500$ ampere-turns per inch in length. But there are $\frac{2600}{14} = 185.7$ turns per inch in length. Therefore we will require $\frac{500}{185.7} = 2.69$ amperes.

MAGNETISM IN IRON.

If a bar of iron be placed inside a helix through which a current of electricity is flowing, the number of lines of force produced are enormously increased.

This power which the bar of iron has of, as it were, creating lines of force, is called its magnetic permeability, or simply its permeability.

PERMEABILITY.

The permeability of a piece of iron is expressed numerically by dividing the number of lines of force which pass through it by the number which would pass through the same space if the iron was not there.

Example 4.

A bar of iron is placed in the helix described in example 1 and the number of lines of force per inch passing through it is found to be 50,000; what is its permeability?

Solution:

The number o lines per square inch in the space enclosed by the helix before the iron was there was 76.8. Therefore the permeability is $\frac{50000}{76.8}$ = 651 nearly.

There are only 3 substances which show this high permeability to magnetic force, viz: iron, nickle and cobalt. Of these three, iron is far the most important, since the other two have not as yet found any practical application, for beside being much more expensive than iron, their permeability is much lower.

The permeability of iron is much affected by the presence of impurities, such as carbon, phosphorus, etc. Thus steel has less permeability than wrought iron and cast iron has less than either. The permeability of iron and steel also decreases as the number of lines per square inch in it increases, or, as it is commonly expressed, as it approaches saturation. This decrease in permeability with increase in induction does not follow any regular law, however, and indeed is different with every sample of iron.

THE SATURATION CURVE.

To find the permeability of iron at different inductions, a curve called a saturation curve of the iron is used. This curve is plotted from a number of tests made on a piece of iron and is only good for the particular piece of iron on which the tests

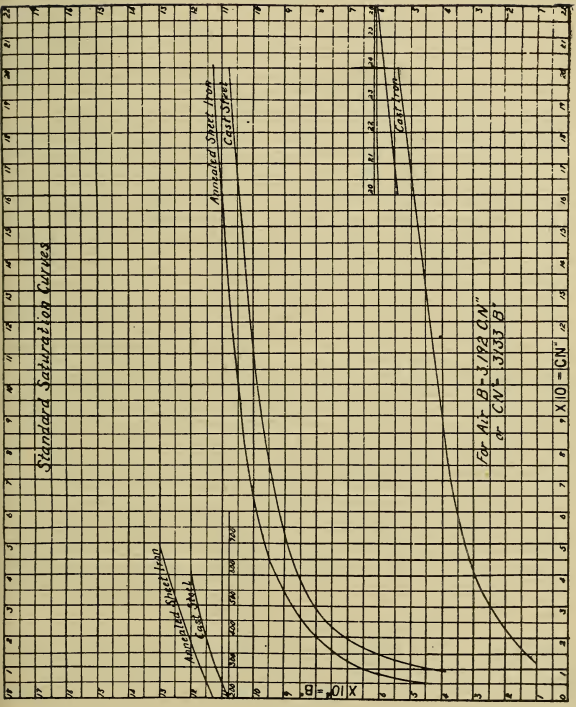
Standard Saturation Curves
Annealed Sheet Iron
Cast Steel
Annealed Sheet Iron
Cast Steel
Cast Iron
For Air B = 3.192 CN'
or CN' = .3133 B'
X10 = B
X10 = CN'

FIGURE 2.

were made from which the curve was plotted. The curves shown in figure 2 represent the average of a large number of tests made with many different samples of the metals sheet iron, cast steel and cast iron, and are believed to give a rather fair average value for the iron and steel at present used in the manufacture of electrical machinery.

This curve does not give the permeability of the iron directly, but shows what is more important, the induction per square inch in the iron which will be produced by a certain number of ampere-turns per inch of its length.

Thus if we wish to find how many ampere-turns are necessary to produce a certain induction in an iron ring, we find in the vertical scale of the figure the number of lines per square inch; follow a horizontal line from there to its intersection with the curve, then from this intersection go vertically downward to the horizontal scale. The number found on this scale is the number of ampere-turns required per inch in length of the magnetic circuit in the iron ring.

Rule 29. To obtain the number of ampere-turns required to drive a certain number of lines of force through an iron ring, divide the total number of lines of force by the number of (*square*) *inches cross section of the ring to obtain the induction per square inch. Find on a saturation curve of the iron the number of*

ampere-turns per inch in length corresponding to this induction per square inch. Multiply this number by the average length of the magnetic circuit in the iron ring in inches. The result is the number of ampere turns required.

EXAMPLE 5.

A cast iron ring has a circular cross section $1\frac{1}{2}$ inches in diameter. The length of the axis of the cylindrical ring is 36″. How many ampere-turns will be required to drive 50,000 lines of force through it? Suppose there are 450 turns on it, how many amperes must be driven through the coil?

Solution:

The square inches cross-section of the ring are $1.5^2 \times .7854 = 1.767$. The lines per square inch $= \frac{50000}{1.767} = 28{,}300$. From figure 2, the ampere-turns per inch in length equal 36. $36 \times 36 = 1296$ = the number of ampere-turns required.

If there are 450 turns on the ring, we will require $\frac{1296}{450}$ amperes $= 2.88$.

MAGNETIZING FORCE.

The number of lines of force per square centimeter produced by a coil of wire carrying a current in air is called "Magnetizing force."

Magnetizing force per square centimeter or per square inch, as well as ampere-turns, is also called "Magneto-Motive-Force" (M. M. F.)

Saturation curves are often given in terms of lines per square centimeter and magnetizing force.

Rule 30. To find the permeability of a piece of iron at any point on the saturation curve, divide the number of lines of force per square centimeter by the magnetizing force per square centimeter. The result is the permeability of the iron at that saturation.

If the curve is given in terms of lines per square inch and ampere-turns per inch in length.

Rule 31. Multiply the ampere-turns per inch in length by 3.2 to find the magnetizing force per square inch and divide the induction per square inch by this product. This result is the permeability.

Rule 32. To reduce from lines of force per square inch to lines of force per square centimeter, divided by 6.45.

Rule 33. To reduce from lines of force per square centimeter to lines of force per square inch, multiply by 6.45.

Rule 34. To reduce from ampere-turns per inch in length to magnetizing force per square centimeter, multiply by 3.2 and divide by 6.45.

Rule 35. To reduce from magnetizing force per square centimeter to ampere-turns per inch in length, divide by 3.2 and multiply by 6.45.

Note.—In most of the electrical books and electrical journals of the present day, Saturation curves are given in terms of lines of force per square centimeter and magnetizing force per square centimeter. The conventional letter used to represent the former is capital B, either in block letter or old English and capital H is used to represent the latter. The rules just stated were given to enable the student to reduce such curves to lines per square inch and ampere-turns per inch in length and *vice versa*.

Example 6.

A wrought iron ring has 10,000 lines of force per square centimeter traversing it. How many ampere-turns per inch in length are required to produce this induction?

Solution:

By rule 33 the induction per square inch equals $10{,}000 \times 6.45 = 64{,}500$. From figure 2 the ampere-turns per inch in length $= 9.5$.

Example 7.

What is the permeability (a) of iron at 80,000 lines per square inch? (b) of cast steel at the same induction? (c) of cast iron at 40,000 lines per square inch?

Solution:

(a) The ampere-turns per inch in length equal

20. By rule 31 magnetizing force per square inch $= 20 \times 3.2 = 64$ and permeability $= \frac{80,000}{64} = 1,250$.

(b) Ampere-turns per inch in length $= 29$. Magnetizing force per square inch $= 29 \times 3.2 = 92.8$ and permeability $= \frac{80,000}{92.8} = 862$.

(c) The ampere-turns per inch in length $= 88$. By rule 31 magnetizing force per square inch $= 88 \times 3.2 = 281.6$ and permeability $= \frac{40,000}{281.6} = 142$.

EXAMPLE 8.

(a) What magnetizing force per square centimeter is required to drive 105,000 lines per square inch through iron? (b) How many lines per square centimeter is 105,000 per square inch? (c) What is the permeability of the iron at that induction?

Solution:

(a) From figure 2 the ampere-turns per inch in length $= 100$. $100 \times \frac{3.2}{6.45} = 49.6 =$ the magnetizing force per square centimeter by rule 34.

(b) $\frac{105,000}{6.45} = 16,280 =$ lines of force per square centimeter by rule 32.

(c) The ampere-turns 100 multiplied by 3.2 $= 320$. $\frac{105,000}{320} = 328 =$ permeability.

CALCULATION OF AMPERE-TURNS IN DYNAMOS.

If the magnetic circuit consists partly of iron and partly of air, as in dynamo machines, it is necessary to obtain the ampere-turns for each part separately.

Thus for each part of the magnetic circuit, we obtain the induction per square inch and from the saturation curve the ampere-turns per inch in length and multiply this by the length in inches of the part in question. When this process has been followed for all the different parts of the circuit, the ampere-turns thus obtained are all added together and this sum is the number of ampere-turns required to drive the requisite number of lines through the circuit.

When an air gap forms part of the circuit, the number of lines of force per square inch in the gap must be multiplied by .3133 (the reciprocal of 3.2) and by the length of the magnetic circuit of the gap in inches or fractions of an inch.

CO-EFFICIENT OF LEAKAGE.

Also in a dynamo machine it must be remembered that the number of lines in the different parts of the circuit may not all be the same, since some may escape through the air. The part of the circuit in which the lines of force are utilized is in the armature and the number of lines required there is what we start with. Some of the lines which go through the field poles and yoke, leak across through the air without going through the armature. The percentage which thus leaks across varies in different kinds of machines, and the ratio of the lines going through the armature to those

going through the field is called the co-efficient of leakage. This co-efficient has to be determined for every style of machine by experiment or else guessed at more or less accurately by comparison with that of other machines as near like it as can be obtained.

EXAMPLE 9.

Lines through the armature, 1,250,000.
Armature cross section, 12.5 inches.
Armature magnetic length, 6 inches.
Gap cross section, 16 inches.
Length magnetic (2×.125) .25 inches.
Leakage co-efficient, .85.
Field poles cross section, 15 inches.
Field poles magnetic length, 36 inches.
Yoke cross section, 28 inches.
Yoke magnetic length, 12 inches.
Material of armature, sheet iron.
Material of field poles, cast steel.
Material of yoke, cast steel.
How many ampere-turns will be required?

Solution:

Induction per square inch in armature $= \frac{1,250,000}{12.5} = 100,000$.

Ampere-turns per inch, (from figure 2) =63.

Ampere-turns for armature, 63×6=378.

Induction gap $\frac{1,250,000}{16} = 78,250$.

78,250 × .3133= 24,500=ampere-turns per inch in length.

Ampere-turns for the gap=24,500 × .25=6,125. The length in the gap is $\frac{1}{8}$ inch on each side and as there are 2 gaps, the total magnetic length is the sum of the 2 or $\frac{1}{4}$ inch.

The total number of lines passing through the field poles $= \frac{1,250,000}{.85} = 1,470,000$.

Induction per square inch in field poles $= \frac{1,470,000}{15} = 98,000$.

From figure 2, 93 ampere-turns are required per inch.

Total ampere-turns for field poles $93 \times 36 = 3,350$.

Induction in yoke $= \frac{1,470,000}{28} = 52,500$.

Ampere-turns per inch, 12. Ampere-turns in yoke $12 \times 12 = 144$, and we have:

Ampere-turns required for armature,	378
Ampere-turns required for gap,	6,125
Ampere-turns required for poles,	3,350
Ampere-turns required for yoke,	144
Total,	9,997

That is: we must have 10,000 ampere-turns in the field coil of this machine in order to drive 1,250,000 lines of force through the armature.

Example 10.

Lines through the armature, 2,740,000.

Armature cross section, 50 inches.

Armature magnetic length, 5.5 inches.

Gap cross section, 95.6 inches.
Gap magnetic length, .25 inches.
Leakage co-efficient, .85.
Field poles cross section, 115 inches.
Field poles length 7 inches.
Yoke cross section 80 inches.
Yoke length 6 inches.
Material of armature, sheet iron.
Material of field poles, cast iron.
Material of yoke, cast iron.
How many ampere-turns will be required?

Solution:

Induction per square inch in armature $= \frac{2,740,000}{50} = 54,800$.

Ampere-turns per inch, (from figure 2) $= 7$.

Ampere-turns for armature $= 5.5 \times 7 = 38.5$.

Induction in the gap $= \frac{2,740,000}{95.6} = 28,600$.

$28,600 \times .3133 = 8,960$ = ampere-turns per inch in length.

$8,960 \times .25 = 2,240$ = ampere-turns in the gap.

The total number of lines passing through the field poles $= \frac{2,740,000}{.85} = 3,220,000$.

Induction in field poles $= \frac{3,220,000}{115} = 28,000$.

From figure 2 ampere-turns per inch in length $= 36$.

Total ampere-turns for field poles $36 \times 7 = 282$.

Induction in yoke $= \frac{3,220,000}{80} = 40,500$.

Ampere-turns per inch 92. Ampere-turns in yoke $6 \times 92 = 552$, and we have:

Ampere-turns required for armature,	38.5
Ampere-turns required for gap,	2,240.0
Ampere-turns required for field poles,	282
Ampere-turns required for yoke,	552
Total,	3,112.5

That is: we must have 3,100 ampere-turns in the field coil of this machine in order to drive 2,740,000 lines of force through the armature.

MULTIPOLAR DYNAMOS.

In the case of a multipolar dynamo (one with more than 2 poles) all the lines of force thread 2 field coils. Each coil therefore only drives the lines through half the total length of the magnetic circuit. Also the lines, after passing through the field pole and entering the armature, divide through two paths, half going in one direction and half in the opposite direction through the armature. They then enter and pass through the two adjacent poles, together with an equal number of lines coming from the armature on the other side of these poles. On reaching the yoke, they separate from these lines again and passing through the yoke, rejoin the lines they started with, as they enter the pole. Thus the number of lines threading the part of the armature which is between the two poles is half the number threading the poles, and the

number passing through the yoke is also half the number passing through the poles.

Rule 36. To find the ampere-turns per coil on a multipolar armature having one coil per pole, find the ampere-turns required to drive the lines through one pole, one gap and the armature and yoke as far as half the distance to the next pole. The sum of these quantities is the ampere-turns per coil.

Note.—If there is only one coil to 2 poles, the number obtained by this rule must be multiplied by two.

Example 11.

An iron clad armature is one in which the armature conductors are embedded in slots cut in the periphery of the armature and the iron parts between the conductors are called "armature teeth."

Total number of lines from one pole passing through the teeth, 6,920,000.

Leakage co-efficient, .87.

Cross section of teeth under one pole 62 inches.

Length of teeth 1.36 inches.

Material of teeth is sheet iron.

Cross section of gap 138 inches.

Length of gap (clearance) =.09375 inches.

Armature core cross section 75.25 square inches.

Armature core length 6.5 inches.

Armature core material, sheet iron.

Field pole cross section 155 square inches.

Field pole length 10.31 inches.

Field pole material, cast iron.

Yoke cross section 118.75 inches.

Yoke length 10.65 inches.

Yoke material, cast iron.

Find the ampere-turns per coil with one coil per pole.

Solution :

Density in the teeth $= \frac{6,920,000}{62} = 11,200$ lines.

The term density is commonly used for the number of lines per square unit.

Ampere-turns per inch in length (from figure 2) $= 190$.

Ampere-turns in the teeth $190 \times 1.36 = 258$.

Density in the gap $= \frac{6,920,000}{138} = 50,000$.

Ampere-turns $= 50,000 \times .21 \times .09375 = 985$.

Density in armature core $= \frac{3,460,000}{75.25} = 45,800$.

Ampere-turns in armature core $= 6 \times 6.5 = 40$.

The term total flux is commonly used for total number of lines of force.

Total flux through field pole $= 7,950,000$.

Density in the field pole $= \frac{7,950,000}{155} = 51,400$.

Ampere-turns $= 10.31 \times 184 = 1,890$.

Density in the yoke, $\frac{3,975,000}{118.75} = 37,000$.

Ampere-turns $= 578$.

Total ampere-turns $= 985 + 256 + 40 + 1,890 + 578 = 3,750$.

Example 12.

Assume a multipolar dynamo with an iron clad armature.

Total flux through the teeth from one pole, 20,450,000.

Leakage co-efficient .89.

Cross section of teeth under one pole 151.5 square inches.

Length of teeth 1.75 inches.

Material, sheet iron.

Gap cross section 538 square inches.

Gap length .25 inches.

Armature core cross section 124.3 square inches.

Armature core length 14 inches.

Armature core material, sheet iron.

Field pole cross section 237.2 square inches.

Field pole length 16 inches.

Field pole material, cast steel.

Yoke cross section=132 square inches.

Yoke length=28.5 inches.

Yoke material, cast steel.

Find the ampere-turns per coil with one coil per pole.

Solution :

Density in the teeth$=\frac{20,450,000}{151.5}=135,000.$

Ampere-turns $=555+1.75=972.$

Density in the gap $=\frac{20,450,000}{538}=38\,000.$

Ampere-turns $= 38{,}000 \times .21 \times .25 = 1{,}995.$

Density in the armature core $= \frac{10{,}225{,}000}{124.3} = 82{,}300.$

Ampere-turns $= 22 \times 14 = 308.$

Total flux through field pole $\frac{20{,}450{,}000}{.89} = 23{,}000{,}000$

Density in the field pole $= \frac{23{,}000{,}000}{237} = 97{,}000.$

Ampere-turns in the field pole $= 1{,}365.$

Density in the yoke $= \frac{11{,}500{,}000}{132} = 87{,}200,$

Ampere-turns $= 42.5 \times 28.5 = 1{,}210.$

Total ampere-turns $= 972 + 1{,}995 + 308 + 1{,}365 + 1{,}210 = 5{,}850.$

TRACTIVE FORCE.

Rule 37. To find the force in pounds required to pull apart 2 magnets, which are in contact, multiply the square of the induction per square inch by the number of square inches of contact and divide by 72,134,000.

Rule 38. To find the number of lines per square inch, the pull in pounds being known, multiply the pull in pounds by 72,134,000, divide by the area of contact in inches and extract the square root.

EXAMPLE 13.

A bar of iron is magnetized to 60,000 lines per square inch and has an area of 12 square inches. How many pounds weight can it sustain?

Solution:

$$\text{Pounds} = \frac{3,600,000,000}{72,134,000} \times 12 = 600 \text{ nearly.}$$

Example 14.

A magnet with 3 inches cross section sustains 24 pounds weight. What is the induction per square inch?

Solution:

$$\text{Induction per square inch} = \sqrt{\frac{24 \times 72,134,000}{3}}\ 2 = 4,000.$$

CHAPTER VI.

DIRECT CURRENT DYNAMOS AND MOTORS.

There are two factors which usually limit the energy which a dynamo is capable of delivering. The one is the speed at which it may be run, which limits the E. M. F. it is capable of generating. The other its capacity of radiating the heat which is developed in its coils, in the iron of the armature, and in some cases, in the iron of its field poles.

The armature is that part of the dynamo in which the E. M. F. is generated and which furnishes current to the outside circuit.

The field magnet, or simply the field, is that part of the dynamo which serves to complete the magnetic circuit of the lines of force which traverse the armature and which also serves as a support for the field coils.

The field coils are coils of wire through which a current is made to circulate, which furnishes the magneto-motive-force required to drive the lines of force through the armature.

In almost all dynamos the armature rotates and the field is still. In a very small proportion of them the field rotates and the armature is still.

The limiting speed of armatures depends mainly on their diameter; the greater the diameter the slower the machine must run. The factor which is usually taken to limit the speed is the surface speed of the armature. This should never be greater than 8,000 feet per minute and in most cases should be very much less. 3,000 feet is a very fair speed for armatures in which the conductors are on the surface.

HEATING OF DYNAMOS.

The temperature to which a dynamo will be raised depends on the amount of heat developed and the radiating surface available for its diffusion.

Rule 39. The rise in temperature in degrees centigrade of any part of a dynamo varies as the number of watts developed in that part of the machine per square inch of the radiating surface

or

The rise in temperature of any part of a dynamo equals a constant multiplied by the watts per square inch of radiating surface.

This constant depends on many things such as the depth of the part, the heat conductivity of the material of which it is composed, the facilities for free circulation of air and many other things.

In ordinary field magnet coils, where the depth of the coil is not more than three inches and its

length across the wires is great compared with its depth, 80 may be taken as a fairly conservative value for this constant, which means that for every watt per square inch of radiating surface, the temperature will rise 80° centigrade. By the radiating surface is meant the surface of the wires themselves. The ends of the coil are not taken into account.

Example 1.

A field coil has a resistance of 1 ohm, and a current of 12 amperes flows through it.

It is 12 inches long and the diameter of its radiating surface is 12 inches. How much will it rise in temperature?

Solution:

Watts $= 12^2 \times 1 = 144$.

Radiating surface $= 12 \times 3.142 \times 12 = 452$.

Watts per square inch $= \frac{144}{452} = .319$.

Rise of temperature in degrees centigrade $= 80 \times .32 = 25.6$.

Example 2.

A field coil has 6,000 ampere-turns, 450 turns, resistance .47 ohms, length across wires $8\frac{1}{4}$ inches, diameter of surface 12 inches. Find the watts per square inch and the probable rise in temperature.

Solution:

Current $= \frac{6,000}{450} = 13\frac{1}{3}$.

Watts $= 13\frac{1}{3}^2 \times .47 = 177 \times .47 = 83.$

Radiating surface $= 12 \times 3.142 \times 8\frac{1}{4} = 311.$

Watts per square inch $= \frac{83}{311} = 26.7.$

Increase in temperature $= 80 \times .27 = 21.6$ degrees.

The temperature constant varies very widely in armatures since it depends on the surface speed of the armature, on the facilities for ventilation and on many other things. It is of course much less in armatures which are turning at a high speed in the atmosphere than in field coils, which are still. A value which has been used under favorable conditions is 11^0 centigrade per watt per square inch, but the value sometimes runs as high as 20 or 30 or even higher if the armature be not well ventilated and the surface speed is low.

ELECTROMOTIVE FORCE OF DYNAMOS.

Rule 40. The E. M. F. of a direct current 2-pole armature is obtained by multiplying together the revolutions per second, the total number of external conductors in series all around the armature and the total flux from one pole through the armature and dividing by 10^8 (100,000,000.)

EXAMPLE 3.

Revolutions per second = 20.

Conductors in series 500.

Flux = 1,000,000.

What is E. M. F. developed?

Solution:

E. M. F. $= \frac{10^8 \times 20 \times 500}{10^8} = 100$ volts.

EXAMPLE 4.

Revolutions per minute 1,500.
Conductors in series 340.
Flux 3,500,000.
What is the E. M. F. developed?

Solution:

E. M. F. $= \frac{3{,}500{,}000 \times 1{,}500 \times 340}{10^8 \times 60} = 297.5$.

Rule 41. The flux required to produce a certain E. M. F. with a certain speed and number of conductors, equals the E. M. F. multiplied by 10^8 *and divided by the revolutions per second and by the number of conductors in series around the external surface of the armature.*

EXAMPLE 5.

Number of conductors 480.
Revolutions per second 23.
Electromotive force 120.
What is the flux?

Solution:

Flux $= \frac{120 \times 10^8}{480 \times 23} = 1{,}100{,}000$.

EXAMPLE 6.

Number of conductors 3,200.

Revolutions per second 33.
Electromotive force 1,000.
What is the flux?
Solution:

$$\text{Flux} = \frac{1{,}000 \times 10^8}{3{,}200 \times 33} = 947{,}000.$$

FREQUENCY.

In the case of multipolar machine, each conductor cuts across the lines of force from each pole as many times in a revolution as there are poles. In a two-pole machine, each conductor cuts the lines of force twice in a revolution, once as it passes the north pole and once as it passes the south pole. A revolution in a two-pole machine is therefore equivalent in a multipolar machine to a movement of any point on the periphery of the armature past two poles. The time in seconds required for any point in the periphery of the armature to pass two poles is called its "period" and the number of these periods in a second is called its "frequency."

Rule 42. To find the frequency of any machine, multiply the revolutions per second by one-half the number of poles.

Rule 43. To find the number of poles required to produce a certain frequency at a certain number of revolutions per second, divide the frequency by the revolutions per second and multiply by two.

EXAMPLE 7.

What is the frequency of a 20-pole machine running at 1,500 revolutions per minute?

Solution:

Revolutions per second $= \frac{1,500}{60} = 25.$

$25 \times \frac{20}{2} = 250 =$ frequency.

EXAMPLE 8.

What is the frequency of a 24-pole machine running at 600 revolutions per minute?

Solution:

Revolutions per second $= \frac{600}{60} = 10.$

Frequency $= 10 \times \frac{24}{2} = 120.$

EXAMPLE 9.

How many poles are required to produce a frequency of 60 in a machine running at 360 revolutions per minute?

Solution:

Revolutions per second $= \frac{360}{60} = 6.$

Poles $= \frac{60}{6} \times 2 = 20.$

EXAMPLE 10.

How many poles are required to produce a frequency of 120 at 1,200 revolutions per minute?

Solution:

Revolutions per second $= \frac{1,200}{60} = 20.$

Poles $= \frac{120}{20} \times 2 = 12.$

ELECTROMOTIVE FORCE IN MULTIPOLAR DYNAMOS.

The connections in multipolar dynamo armatures may be made in two ways. They may be made so that the E. M. F's generated in the coils between the different sets of brushes are added together, or so that the currents in these different circuits are added together.

The first method is called the "series" winding, the second the "multiple" winding.

A multipolar dynamo is the equivalent of as many two-pole dynamos as there are pairs of poles, each two-pole dynamo having as many conductors on its surface as there are all around the armature of the multipolar dynamo, and having for its revolutions per second the frequency of the multipolar dynamo.

The E. M. F. between each pair of brushes is then the same as it would be in a two-pole machine whose revolutions per second are equal to the frequency of the multipolar machine, and having a number of conductors on its armature equal to the total number on the multipolar machine divided by half the number of poles. It must be remembered that the flux assumed in this reasoning is that from one pole. We have then:

Rule 44. The E. M. F. of a multipolar machine with a series winding equals the frequency multiplied

by the total number of conductors all around the periphery of the armature multiplied by the flux from one pole divided by 10^8.

Rule 45. The flux required to produce a certain E. M. F. with the number of poles and conductors and the frequency fixed, with series winding is equal to the E. M. F. times 10^8 *divided by the frequency times the total number of conductors.*

Rule 46. The E. M. F. of a multipolar machine with a multiple winding equals the revolutions per second, times the total number of conductors, times the flux from one pole divided by 10^8.

Rule 47. The flux required to produce a certain E. M. F. with the number of conductors and poles and the frequency fixed, with multiple winding equals 10^8 *times the E. M. F. divided by the revolutions per second, times the number of conductors.*

Example 11.

A 10-pole multipolar dynamo has a speed of 500 revolutions per minute, flux 8,200,000, number of conductors 580, series wound.

What is the E. M. F?

Solution:

Revolutions per second $= \frac{500}{60} = 8\frac{1}{3}$.

Frequency $= 5 \times 8\frac{1}{3} = 41\frac{2}{3}$.

$E = \frac{41\frac{2}{3} \times 580 \times 8{,}200{,}000}{10^8} = 1{,}982$ volts.

Example 12.

An 8-pole multipolar, multiple wound, has a speed of 150 revolutions per minute, flux 7,900,000, number of conductors 320. What is the E. M. F?

Solution:

Revolutions per second $=\frac{150}{60}=2.5$.

$E=\frac{2.5\times320\times7,900,000}{10^8}=63.2$ volts.

Example 13.

A 6-pole multipolar, with 186 conductors, series wound, on the armature is required to give 100 volts potential at 360 revolutions per minute. What is the flux required?

Solution:

Revolutions per second $=6$.

Frequency $=6\times3=18$.

Flux $=\frac{100\times10^8}{18\times186}=3,000,000$ nearly.

Example 14.

A 4-pole multipolar with 348 conductors, multiple wound, is required to give 500 volts potential at 530 revolutions per minute. What flux per pole is required?

Solution:

Revolutions per second $=\frac{540}{60}=9$.

Flux $=\frac{500\times10^8}{9\times348}=16,000,000$ nearly.

EFFICIENCY.

Two classes of efficiency are distinguished, *Electrical* and *Commercial.*

Electrical efficiency is the ratio of the electrical energy delivered to the total electrical energy of the machine. The total electrical energy of the dynamo is equal to the electrical energy delivered plus the electrical energy wasted in the machine.

There are in general but two sources of loss of electrical energy in a dynamo, the one C^2 R loss in the armature coils, the other C^2 R loss in the field coils. Eddy currents and hysteresis loss in the iron are not commonly included in the electrical losses.

Rule 48. To find the electrical efficiency of a dynamo, add together the electrical energy delivered the loss in the armature coils and the loss in the field coils and divide the energy delivered by this sum and multiply by 100.

EXAMPLE 15.

Energy delivered 500 kilowatts, lost in the armature coils 12 kilowatts; lost in the field coils 8 kilowatts. What is the electrical efficiency?

Solution:

$$\text{Efficiency} = \frac{500 \times 100}{500+12+8} = 96\ \%.$$

Electrical efficiency is of little use to the practical man or in fact to any one except as a matter

of scientific interest. Commercial efficiency is far more important and of direct practical use.

Commercial efficiency is the ratio of the energy delivered by the dynamo to the total energy delivered to the dynamo. The mechanical losses must therefore be added to the electrical losses in getting the commercial efficiency. Mechanical losses include friction in the bearings, hysteresis and eddy current loss in the armature core and in some cases hysteresis and eddy current losses in the field poles.

Rule 49. To obtain the commercial efficiency of a dynamo, add together the energy delivered, the $C^2 R$ loss in the armature and field coils, the friction loss in the bearings, the hysteresis and eddy current loss in the armature core, and the hysteresis and eddy current loss in the field poles and divide the energy delivered by this sum and multiply by 100.

Example 16.

Energy delivered 30 kilowatts, loss in armature coils .6 kilowatts, loss in field coils .7½ kilowatts, hysteresis and eddy currents in the armature core .4 kilowatts. (This includes the loss in the field poles), friction of bearings 1 kilowatt. What is the commercial efficiency?

Solution:

Total energy delivered to the machine=30+ .6 +.7, 5+.4+1=32.75

Commercial efficiency= $\frac{30\times100}{32.75}$= 91.6.

Example 17.

Energy delivered 300 kilowatts, loss in the armature coils 2.6 kilowatts, loss in the field coils 2.2 kilowatts. Hysteresis and eddy current loss in the armature core 3.2 kilowatts, friction loss 4.5 kilowatts. What is the commercial efficiency?

Solution:

Total energy delivered to the machine = 300-+ 2.6+2.2+3.2+4.5=312.5.

Commercial efficiency = $\frac{300\times100}{312.5}$ = 96%.

Note.—It must be remembered that the E.M.F. of dynamos as given by the above rules is that which is generated by the lines of force actually threading the armature coils and also that all of this E. M. F. is not necessarily available at the terminals of the machine. In fact the E. M. F. at the brushes is always less than that generated in the coils on the armature, except on open circuit, that is, when no current is being taken from the armature.

When current is being taken from the armature some of the E. M. F. is taken up in driving the current through the armature coils. The amount of this is determined simply by Ohm's law, by multiplying the current flowing through the armature by its resistance.

Rule 50. The E. M. F. at the brushes of a dynamo is equal to the total E. M. F. generated in the armature coils minus the product of the current in the armature by its resistance.

If there is a series coil on the field, that is, a coil through which the whole armature current circulates, the resistance of this series coil must be added to that of the armature.

EXAMPLE 18.

The total E. M. F. of an armature is 1,100 volts, current 50 amperes, resistance 2 ohms. What is the E. M. F. at the brushes?

Solution:

$1{,}100 - 50 \times 2 = 1{,}000$ volts.

EXAMPLE 19.

The total E. M. F. of an armature is 130 volts, current 500 amperes, resistance .04 ohms. What is the E. M. F. at the brushes?

Solution:

$135 - .04 \times 500 = 115$ volts.

Another element which must be carefully considered is what is usually called armature reaction. The coils on the armature being traversed by a current and surrounding a part of the magnetic circuit of the machine have their effect in producing lines of force. The effect of the currents in the armature coils is mainly to prevent the line of force

produced by the field coils from entering the armature. The amount of this effect is dependent on so many conditions and is so little understood at the present day, that it is not possible to give any simple rule by which to calculate it. It is, however, very much more important in very large machines than it is in small ones. In small machines the armature reaction may be neglected as compared with the loss of E. M. F. due to resistance. In very large machines, however, the resistance is so low that the loss of E. M. F. by resistance may be neglected compared with the loss due to armature reaction.

DIRECT CURRENT MOTORS.

The counter—E. M. F. of a motor is the E. M. F. generated in its armature coils by the revolution of the armature. It always opposes the passage of the current flowing through the armature coils. The rules for obtaining this E. M. F. are the same as those for obtaining the E. M. F. of direct current dynamos. The impressed E. M. F. is that of the line from which current is taken to drive the motor.

Rule 51. The current going through the armature coils of a direct current motor equals the difference of the impressed and counter E. M. F. divided by the resistance of the armature circuit.

Rule 52. The mechanical work done by a motor

equals the current going through its armature coils multiplied by the counter—E. M. F. of the motor.

EXAMPLE 20.

Impressed E. M. F. 110 volts.

Counter E. M. F. 90 volts.

Resistance 1.5 ohms.

What is the current flowing through the armature coils and the work done by the motor?

Solution:

$C=\frac{110-90}{1.5}=13\frac{1}{3}$.

Watts work done by the motor$=13\frac{1}{3}\times 90=1{,}200$.

EXAMPLE 21.

In the last example, find the work done by the motor when the counter E. M. F. has the value (1) 80, (5) 70, (3) 55, (4) 40.

Solution:

(1) current$=\frac{110-80}{1.5}=20$ work$=20\times 80=600$ watts.

(2) current$=\frac{110-80}{1.5}=26\frac{2}{3}$ work$=26\frac{2}{3}\times 70=$ 1,866.

(3) $\frac{110-55}{1.5}=36\frac{2}{3}=$ current work$=36\frac{2}{3}\times 55=$ 2,017.

(4) current$=\frac{110-40}{1.5}=46\frac{2}{3}$ work$=46\frac{2}{3}\times 40=$ 1,867.

In this example it will be noticed that as the

counter E. M. F. decreases, the work done increases until the counter E. M. F. reaches 55 volts, which is one-half the impressed E. M. F. As the counter E. M. F. decreases beyond this the work done begins to decrease. This is the general law of direct current motors, that the maximum work is done when the counter E. M. F. equals $\frac{1}{2}$ the impressed E. M. F.

EFFICIENCY OF DIRECT CURRENT MOTORS.

The efficiency of a motor, like that of a dynamo, is the ratio of the output of the motor to the input. In the case of the motor, we usually know the energy supplied by the line and the losses in the motor have to be substracted from this to obtain the available energy. These losses are of 2 classes, electrical and mechanical, the electrical being represented by the C^2 R losses in the armature and field coils, the mechanical by the friction losses in the bearings and the hysteresis and eddy current losses in the armature core. The work which is represented by the product of the armature current and counter E. M. F. of the motor, includes the mechanical losses. The energy supplied to the motor equals the product of the impressed E. M. F. and the current, the impressed E. M. F. being the E. M. F. at the terminals of the motor.

Example 22.

A motor delivering 15-horse power has a resis-

tance of 2.34 ohms, core loss 1,100 watts, mechanical losses 1,300 watts, impressed E. M. F. 500 volts, current 32 amperes.

Solution:

Watts delivered $=15\times746=11{,}200$.

C^2 R loss $= 32^2\times2.34=2{,}400$.

Core loss is another name for the hysteresis and current loss in the armature core.

Watts delivered to the motor $11{,}200+2{,}400+1{,}100+1{,}300=16{,}000$.

Efficiency $=\frac{11{,}200}{16{,}000}=.70$ or 70%.

CHAPTER VII.

ALTERNATING CURRENT APPARATUS — DEFINITION OF ALTERNATING CURRENT.

Alternating currents are currents which reverse their direction of flow periodically at very short intervals of time. The change is usually not abrupt, but starting from zero the current gradually grows in strength to a certain maximum value, then gradually decreases to zero, then commences to increase again to a negative maximum, decreases to zero and starts through the cycle of operations again.

The time from one positive maximum to the next positive maximum is called the *period* of that current. There are two alternations in each period, one from positive to negative and one from negative to positive.

The number of periods in one second is called the *periodicity* or *frequency*. These terms are also sometimes used to denote the number of periods per minute, but the periods per second is far preferable. We will, in what follows, use the term frequency to denote periods per second.

It will be noted that we used the term frequency in dealing with direct current machines in

apparently a different sense. The difference is more apparent than real, however, since the frequency of any alternator is obtained in the same manner as that of the multipolar direct current dynamo, namely by multiplying the revolutions per second by $\frac{1}{2}$ the number of poles.

Alternating currents are produced by alternating electro-motive forces which vary in the same way as the currents do.

The strength of an alternating current is measured by the amount of heat energy it will develop in traversing a resistance. The other attributes which serve as a basis of measurement with direct currents are not available with alternating currents, except by means of subsidiary appliances, which it would be outside the scope of this book to discuss.

The strength of the alternating current based on its heat-producing power, bears a fixed ratio to its maximum value, this ratio depending on the law of variation of the current. Most alternators of the present day however, deliver an E. M. F. and therefore a current which varies with time very nearly as the sine of an angle does with the angle, and it is almost universally assumed that the alternating current, and therefore the alternating E. M. F. varies according to this law.

The alternating current is said to have a strength of one ampere when the heat devoloped by it in

passing through a certain resistance is the same as would be developed by a direct current of one ampere strength passing through the same resistance. This value may be algebraically proved to be the square root of the mean of the squares of all the instantaneous values of the current throughout one complete period, or as it is more shortly expressed "the square root of the mean square," or more shortly still, "$\sqrt{\text{mean}^2}$."

The ratio of the square root of the mean square of an alternating current to the maximum value is as one to the square root of two. The same statement, of course, also holds true for an alternating E. M. F.

Rule 53. To reduce from the maximum value of an alternating current, or E. M. F. to the square root of the mean square, divide by the square root of 2 (1.414);

or vice versa

To reduce from the square root of the mean square of an alternating current or E. M. F. to the maximum value, multiply by the square root of 2(1.414).

EXAMPLE 1.

An alternator gives a maximum E. M. F. of 150 volts. What is the square root of the mean square?

Solution:

$$\frac{150}{1.414} = 106 \text{ volts.}$$

EXAMPLE 2.

What is the maximum value of (1) a 25 ampere current? (2) a 35 ampere current, and (3), what is the maximum value of the E. M. F. required to drive these currents through a resistance of 60 ohms, and (4), what is the square root of the mean square of this E. M. F.?

Solution:

(1) $25 \times 1.414 = 35$ amperes.

(2) $35 \times 1.414 = 49.5$ amperes.

(3) This solution may be made by first getting the value of the square root of the mean square of the E. M. F. from that of the current and multiplying by 1.414, or it may be obtained more directly from the maximum value of the current by multiplying by the resistance direct, thus:

E. M. F $= 35.35 \times 60 = 2{,}121$ and
E. M. F. $= 49.5 \times 60 = 2{,}969$.

(4) $25 \times 60 = 1{,}500$ and
$35 \times 60 = 2{,}100$.

Hereafter in speaking of alternating currents and electromotive forces, the square root of the mean square is always understood unless it is otherwise stated. Ohm's laws then applies to al-

ternating currents and electromotive forces, just as it does to direct currents unless there is a transformer or choking coil or some other source of high inductance in the circuit. The effect of inductance will be explained later.

Rule 54. The E. M. F. of an alternator is equal to the flux from one pole multipled by the number of face conductors in series multiplied by the frequency, multiplied by the constant 2.22 and divided by 10^8

EXAMPLE 3.

Flux from one pole, 3,700,000.
Face conductors in series, 120.
Frequencey, 125.
What is the E. M. F?
Solution:

$$E = \frac{3{,}700{,}000 \times 120 \times 125 \times 2.22}{10^8} = 1{,}232.$$

EXAMPLE 4.

Flux from one pole, 18,000,000.
Face conductors in series, 300.
Frequency, 30.
What is the E. M. F.?
Solution:

$$E = \frac{2.22 \times 18{,}000{,}000 \times 300 \times 30}{10^8} = 3{,}600 \text{ nearly.}$$

Rule 55. Flux from one pole equals the E. M. F. multiplied by 10^8 *and divided by the product of*

the conductors in series, the frequency and the constant 2.22.

EXAMPLE 5.

Frequency, 50.
Number of face conductors, 386.
E. M. F., 2,500.
What is the flux from one pole?
Solution:

$$\text{Flux} = \frac{2{,}500 \times 10^8}{386 \times 50 \times 2.22} = 5{,}600{,}000.$$

EXAMPLE 6.

E. M. F., 1,000.
Number of face conductors, 560.
Frequency, 100.
What is the flux?
Solution:

$$\text{Flux} = \frac{1{,}000 \times 10^8}{560 \times 100 \times 2.22} = 800{,}000.$$

E. M. F. OF 3 PHASE MACHINES.

The armatures of 3 phase machines are wound with 3 circuits which may be connected together in various ways. The two ways most commonly employed are called the "star" winding and the "delta" winding. With the delta winding the E. M. F. between the brushes is the same as that developed in each coil in the armature. With the star winding the E. M. F. between brushes is the

square root of 3 times as great as the E. M. F. generated by each armature coil.

The number of conductors in series on the armature is $\frac{1}{3}$ of the total number all around the periphery.

The E. M. F. of a 3 phase generator is usually taken as that between the brushes and it is this E. M. F. which is meant when the E. M. F. of the machine is spoken of.

Rule 56. To find the E. M. F. per armature coil of a 3 phase generator, star connected, divide the E. M. F. between brushes by 1.732 ($\sqrt{3.}$)

or vice versa

To find the E. M. F. between brushes of a 3 phase generator, star connected, multiply the E. M. F. per coil by 1.732.

Rule 57. To find the current per coil of a 3 phase generator, divide $\frac{1}{3}$ of the capacity of the machine in watts by the E. M. F. per coil.

The rules for obtaining the flux on the E. M. F. per coil from the turns, frequency, etc., are the same as for single phase alternators.

The E. M. F. per coil of a 3 phase generator, delta connected, is the same as the E. M. F. between the brushes.

Example 7.

A 3 phase 40 pole machine, star connected, has 240 conductors on the periphery of the armature. Revolutions per minute 125. Flux from one pole 5,150,000. Watts output 500,000.

What is the E. M. F. between the brushes and the current per armature circuit?

Solution:

Frequency $= \frac{125}{60} \times 20 = 41\frac{2}{3}$.

E. M. F. per coil $= \frac{5{,}150{,}000 \times 2.22 \times \frac{1}{3}240 \times 41\frac{2}{3}}{10^8} = 375$.

E between brushes $= 375 \times 1.732 = 650$ nearly.

Watts per coil $= \frac{500{,}000}{3} = 167{,}000$.

Current per coil $= \frac{167{,}000}{375} = 445$ amperes nearly.

Example 8.

A 10 pole, 3 phase, star connected, generator has 540 face wires Revolutions per minute 600. Flux from one pole 6,920,000. Watts capacity 250,000. What is the E. M. F. between brushes and the current per coil?

Solution:

Frequency $= \frac{600}{60} \times \frac{1}{2}10 = 50$.

E per coil $= \frac{2.22 \times 6{,}920{,}000 \times \frac{1}{3}540 \times 50}{10^8} = 1{,}380$.

E between brushes $= 1{,}380 \times 1.732 = 2{,}400$.

Watts per coil $= \frac{250{,}000}{3} = 83{,}300$.

Current per coil $= \frac{83{,}300}{1{,}380} = 60$ nearly.

Example 9.

An 8 pole, 3 phase, star connected, generator has 864 face conductors. Revolutions per minute 750. E. M. F. between brushes 2,500. Watts capacity 120,000. What is the flux and the current per coil?

Solution:

$$\text{Frequency} = \frac{750}{60} \times 4 = 50.$$

$$\text{E. M. F. per coil} = \frac{2,500}{1.732} = 1,440.$$

$$\text{Flux} = \frac{1,440 \times 10^8}{2.22 \times \frac{1}{3} 864 \times 50} = 4,500,000.$$

$$\text{Watts per coil} = \frac{120,000}{3} \times 40,000.$$

$$\text{Current per coil} = \frac{40,000}{1,440} = 28 \text{ amperes nearly.}$$

Inductance.

Inductance is the E. M. F. which is set up in a circuit by a current flowing through it and changing at the rate of one ampere per second.

The number of ampere-turns added or subtracted per second divided by the magnetic resistance (or reluctance) of the circuit, equals the number of lines of force added or subtracted per second.

The reluctance of a magnetic circuit corresponds to the term resistance in an electric circuit and is equal to the magneto-motive-force divided by the induction. Either of these quantities may be expressed in whatever units may be most convenient.

Adhering to the units we have used throughout

this book, reluctance will equal the ampere-turns per inch in length divided by the lines per square inch.

The number of turns which will be added or subtracted by a change in the current of one ampere equals the number of turns in the coil.

The E. M. F. produced in each turn of the coil will then equal the number of lines introduced into or taken out of the coil in one second, divided by 10^8, provided the rate of change of the number of lines is uniform. This E. M. F. multiplied by the number of turns in the coil will then equal the total E. M. F.

Rule 58. The inductance of a coil of wire encircling a homogeneous magnetic circuit of constant permeability equals the product of the square of the turns in the coil times the permeability of the material of which the magnetic circuit is composed, times its cross section divided by its length, times the constant 3.2 *divided by* 10^8.

Such a condition can only be realized with a non-magnetic material such as air since the permeability varies in all magnetic materials for different inductions. If the non-magnetic material be a metal, such as brass, eddy currents in the brass will effect the apparent inductance, unless the rate of variation of the current is uniform.

EXAMPLE 10.

What is the inductance of a coil of 600 turns in air with an area of 4 square inches and length of 6 inches?

Solution:

Permeability of air is one.

$$\text{Inductance} = \frac{360{,}000 \times 3.2 \times 4}{6 \times 10^8} = .00768.$$

In the case of iron the inductance varies for every point on the saturation curve and in the above rule we must substitute for 3.2 times the permeability, the slope of the saturation curve.

Rule 59. To find the slope of a saturation curve at any point, draw a tangent to the curve at that point, then, considering this tangent as a saturation curve, find the number of lines which will be added by one ampere-turn per inch. This is the slope of the curve.

Rule 60. The inductance of a coil of wire encircling a homogeneous ring of magnetic material of varying permeability equals the square of the number of turns, times the cross section in inches, times the slope of the saturation curve, divided by the product of the length and 10^8.

If the permeability is constant at all inductions the slope equals 3.2 times the permeability.

Example 11.

What is the inductance of a coil of wire encircling a ring of sheet iron of $3\frac{1}{2}$ inches cross-section and 24 inches long, 540 turns, at 60,000 lines per square inch?

Solution:

Referring to the saturation curve, figure 2 we find that the slope at 60,000 lines per square inch is 28,000. Therefore:

$$\text{Inductance} = \frac{540^2 \times 3\frac{1}{2} \times 28{,}000}{24 \times 10^8} = 12 \text{ nearly.}$$

The name of the unit of inductance is the *Henry* and the inductance in the above example is 12 henrys.

TRANSFORMERS.

A transformer may be considered to be a dynamo in which the change of flux through the armature coils is produced by variation in the field current, instead of by revolution of the armature.

A transformer is made up of 3 parts, a primary coil, a secondary coil, and a magnetic core. The primary coil corresponds to the field coil of a dynamo, the secondary to the armature coil and the core performs its usual function of furnishing a path for the lines of force.

An alternating current sent through this primary coil sets up an alternating magnetism in the core and this changing magnetism sets up an E. M. F.

in both the primary and secondary coil. The E. M. F. set up in the primary coil is in the main opposed to the current flowing through it and therefore the E. M. F. impressed on the primary must be sufficient to overcome this "counter E. M. F. of self induction" as it is called, as well as to drive the current through the resistance of the primary coil.

Rule 61. The E. M. F. set up in either coil of a transformer by variation of the flux through its core equals 4.44, times the frequency, times the number of turns in the coil, times the maximum flux divided by 10^8.

EXAMPLE 12.

A transformer has 680 turns in the primary and 34 in the secondary coil, frequency 133, maximum flux 160,000. What is the E. M. F.?

Solution:

Primary $E = 4.44 \times 133 \times 680 \times 160{,}000/10^8 = 640.$

Secondary $E = 4.44 \times 133 \times 34 \times 160{,}000\ 10^8 = 32.$

It will be noticed that owing to the fact that the flux through one coil goes through both coils, the only difference in the two coils as far as producing E. M. F. is concerned is in the number of turns in them, and we have:

Rule 62. The ratio of the E. M. F. generated in

the primary coil to that generated in the secondary is the same as the ratio of the turns.

Since in most transformers the E. M. F. required to drive the primary current through the primary resistance is very small compared with the counter E. M. F. due to the variation of the flux, we obtain by neglecting it the following rule, which is accurate enough for all practical purposes.

Rule 63. The ratio of the primary impressed E. M. F. to the secondary induced E. M. F. is very nearly equal to the ratio of the primary turns to the secondary turns.

This ratio is called "the ratio of transformation."

EXAMPLE 13.

Ratio of transformation 20, impressed E. M. F. 1,000. What is the secondary E. M. F.?

Solution:

Secondary E. M. F. $= \frac{1000}{20} = 50$ volts.

EXAMPLE 14.

Ratio of transformation 20, secondary E. M. F. 100 and secondary turns 50. What is the primary E. M. F. and turns?

Solution:

Primary E. M. F. $= 20 \times 100 = 2{,}000$.

Primary turns $= 20 \times 50 = 1{,}000$.

Rule 64. The maximum flux through the core equals the E. M. F. times 10^8 divided by the product of the constant 4.44 by the frequency by the number of turns in the coil.

EXAMPLE 15.

Secondary E. M. F. 50, primary turns 750, frequency 50, ratio of transformation 20. What is the flux?

Solution:

Primary E. M. F. $= 50 \times 20 = 1{,}000$.

Flux $= \frac{1{,}000 \times 10^8}{4.44 \times 50 \times 750} = 600{,}000$.

Rule 65. The ratio of the currents in primary and secondary is very nearly the ratio of transformation.

Rule 66. The drop in E. M. F. at the terminals of the secondary of a transformer, if there be no magnetic leakage, equals the secondary current, multiplied by the secondary resistance, plus the primary current, multiplied by the primary resistance, divided by the ratio of transformation.

The drop due to magnetic leakage is not capable of accurate predetermination and no rule can be given. It can only be guessed at by comparison with other transformers of the same size and type.

EXAMPLE 16.

Primary resistance 30 ohms.

Secondary resistance .1 ohms, secondary current 5 amperes, ratio of transformation 20. What is the probable drop?

Solution:

Drop in secondary coil $5 \times .1 = .5$.

Current in primary $\frac{5}{20} = .25$.

Primary drop $\frac{.25 \times 30}{20} = .375$.

Total drop $= .375 + .5 = .875$.

If there is much inductance in the secondary circuit this drop may be increased somewhat. If there are motors in the secondary circuit the drop may be increased or diminished, depending on many conditions too complicated to treat of here.

EFFICIENCY OF TRANSFORMERS.

There are four sources of loss of energy in a transformer, C^2 R loss in the primary, C^2 R loss in the secondary, hysteresis loss in the iron core and eddy current loss in the core. Hysteresis and eddy current loss are usually lumped together and called the core loss.

CORE LOSS.

The hysteresis loss depends on the quality of the iron used, on the maximum magnetization, on the frequency and on the weight of the core.

Rule 67. Hysteresis loss at any degree of magnetization varies directly with the frequency and the

weight of iron, or Hysteresis loss for any sample of iron at a certain degree of magnetization equals a constant multiplied by the frequency, times the weight.

This constant, which is the hysteresis loss per pound per cycle, varies so much in different samples of iron and even in the same iron under different conditions of tempering and so forth, that a curve showing the hysteresis loss in any one sample would be likely to be misleading.

EXAMPLE 17.

What is the hysteresis loss in a transformer whose core weighs 120 pounds, frequency 125, loss per pound per cycle .040?

Solution:

Hysteresis loss$=120\times125\times.040=600$ watts.

The eddy current loss depends on the quality of the iron, on the degree of lamination and the resistance between laminations on the degree of magnetization and on the frequency.

Rule 68. Eddy currents loss varies directly as the square of the frequency and as the square of the magnetization and inversely as the ohmic resistance of the core.

EXAMPLE 18.

The eddy current loss in the core of a transformer was 500 watts. What would it be if the in-

duction was doubled and the frequency halved?

Solution:

Since the loss varies as the square of the induction, we would get 4 times the loss at the same frequency. Therefore at $\frac{1}{2}$ the frequency we would get $\frac{1}{4}$ of this or the loss will be the same as it was originally.

The C^2 R loss in primary and secondary is easily determined and we have:

Rule 69. The efficiency of a transformer equals the energy delivered, divided by the sum of the energy delivered and the losses in the transformer.

EXAMPLE 19.

Energy delivered, 500 watts.

Loss in the secondary coil, 15 watts.

Loss in the primary coil, 20 watts.

Core loss, 30 watts.

What is the efficiency?

Solution:

$$\text{Efficiency} = \frac{500}{500+15+20+30} = \frac{500}{565} = 88\tfrac{1}{2}\,\%.$$

It will be noticed that the hysteresis and eddy current loss remains constant at all loads, while the C^2 R loss varies with the load. If the hysteresis and eddy current loss is heavy, the transformer will therefore have a very low efficiency at light loads. Thus in the above example, the efficiency

at $\frac{1}{4}$ load is $\frac{125}{1+1\frac{1}{4}+30}$ or a little under 80%. Since a transformer is seldom fully loaded, this low efficiency at light loads enters largely into what is called the "all day efficiency" of a transformer.

The limiting factors in its design are the amount of heat it can radiate, the regulation or drop of E. M. F. at the terminals of the secondary, and the all day efficiency.

CHAPTER VIII.

LIGHTING AND POWER.

The two main objects for which electrical energy is transmitted are to supply light by means of lamps and power by means of motors.

ARC LIGHTING.

In arc light systems the lamps on one circuit are all in series. That is, the whole current from the dynamo goes through all the lamps one after the other, and as new lamps are switched in, thus introducing more resistance in the circuit, the dynamo automatically increases its E. M. F. so as to keep the current very nearly constant. The E. M. F. required to drive the current through the resistance of a lamp varies with lamps of different makes, each lamp being designed to work best with a particular value of current. A common value is a current of 10 amperes requiring 45 volts to drive it through the resistance of each lamp.

Rule 70. The E. M. F. required to drive the current through an arc light circuit equals the E. M. F. per lamp, multiplied by the number of lamps, plus the E. M. F. required to overcome the resistance of the connecting lines.

Example 1.

An arc light circuit of 50 lamps, each requiring 45 volts and 10 amperes, resistance of line being 25 ohms. What is the E. M. F. of the dynamo at full load?

Solution:

$50 \times 45 = 2{,}250 =$ E. M. F. required for the lamps.

$10 \times 25 = 250 =$ E. M. F. required for the line.

$2{,}250 + 250 = 2{,}500 =$ volts at terminals of arc light machine.

The line wire is usually chosen of such a size that the loss of energy in it is 10% of that delivered by the dynamo. Thus in the above example the total energy delivered to the line equals C E equals $2{,}500 \times 10 = 25{,}000$ watts, and that in the line equals $250 + 10 \times 2{,}500$ watts or 10% of the total energy delivered to the line.

The watts consumed by each lamp is of course equal to the current times the E. M. F. at the lamp terminals. Thus in the above example the current being 10 amperes and the E. M. F. 45 volts, the watts per lamp are $10 \times 45 = 450$ and 10% being lost in the line, the energy supplied by the dynamo per lamp will be 500 watts.

Incandescent Lighting.

Here all the lamps in one circuit are in parallel. That is, the E. M. F. between the terminals of

each lamp is less than the E. M. F. of the dynamo by the amount of drop in the line, and the current through the dynamo is the sum of the currents through all the lamps.

The circuits are laid out in one of two ways. Either a simple 2 or 3 wire circuit from which currents is taken off at intervals for the lamps singly or in groups, or they are laid out in a network, connection being made to the dynamo through several paths. The second method is too complicated to treat here.

Rule 71. The loss of E. M. F. between any two points on the line is found by multiplying the current in each part of the line between these points by the resistance of that part and adding all these products together.

EXAMPLE 2.

In a circuit supplying 48 lamps, each taking $\frac{1}{2}$ ampere of current at 120 volts at the generator, resistance of the line per foot being .00039, distance to first group of lamps 1,000 feet, second group 200 feet further, third group 50 feet further and fourth group 100 feet further, each group containing 12 lamps. What is the voltage of each group?

Solution:

Resistance to first group $1{,}000 \times .00039 = .39$.

Current = 24 amperes.

Drop in potential from dynamo to first group 9.36 volts.

Resistance between first and second group $200 \times .00039 = .078$.

Current = 18 amperes.

Drop in potential between first and second group $18 \times .078 = 1.4$.

Resistance between second and third group $50 \times .00039 = .019$.

Current = 18 amperes.

Drop between second and third group $.019 \times 12 = .23$ volts.

Resistance between third and fourth group $100 \times .00039 = .039$.

Current = 6 amperes.

Drop between third and fourth group .23.

Total drop from dynamo to last lamp $9.36 + 1.4 + .23 + .23 = 11.22$.

Difference between first and last group of lamps 1.86 volts.

Here again the watts consumed by a lamp are obtained by multiplying the volts at its terminals by the current flowing through it.

The problems in the case of motors on direct current systems are not at all different from those connected with lighting, unless the motor is moving bodily as in the case of street cars.

WEIGHT OF COPPER IN OVERHEAD LINES.

One of two factors usually limits the size of copper conductors in overhead lines. The permissable variation of E. M. F. between no load and full load, and the permissable loss of energy in the line.

In long distance transmission the permissable loss of energy is usually the limiting factor.

Short and compact formulae are often given for obtaining the weight of copper in an overhead line with the object of saving time in making these calculations, but we consider them mischievous in their tendency, for the reason that mistakes are easily made and easily overlooked.

It is far better to go one step at a time unless one has a large number of such calculations to make in a day, which is seldom the case.

Since the loss of energy through any resistance varies as the square of the current, we have:

Rule 72. The energy lost in a line of a certain resistance, delivering a certain amount of energy varies inversely as the square of the E. M. F.

In other words, doubling the E. M. F. at which the energy is supplied to the line, reduces the loss in the line to one-quarter, and multiplying the E M. F. by 3 reduces the loss to one-ninth.

Rule 73. To find the weight of copper required by a line delivering a certain amount of energy at a

certain percentage of loss in the copper and at a certain voltage.

First. Divide the watts delivered by the voltage to obtain the current.

Second. Divide the permissable loss in the line, by the square of the current, to obtain the resistance of the line.

Third. Divide the resistance of the line, by the number of feet in length of the line, (twice the distance of transmission with an ordinary 2 wire circuit), to obtain the resistance per foot from which the size wire can be obtained by reference to table 6.

Fourth. Multiply the weight of wire per foot by the number of feet of line. The result is the weight of wire in the line.

In the first section of the rule care must be taken not to divide the watts at the receiving end of the line by the volts at the generating end. The watts at the receiving end must be divided by the volts at the receiving end, or the watts at the generating end, by the volts at the generating end.

In the third section, if the resistance per foot should come out lower than that of any wire in the table, more than one wire must be used, the resistance of any number of wires in parallel being equal to the resistance of one of them divided by

their number, (provided they are all of the same size.)

Again some wire gauge tables give the resistance per mile, in which case the total resistance of the line would be divided by the number of miles of wire.

Example 3.

500,000 watts has to be transmitted a distance of 10 miles, with a loss of 20% at a voltage of 5,000 at the receiving end. What is the weight of copper required?

Solution:

Doubt sometimes arises as to whether this means that 500,000 watts are to be fed into the line, the other end of the line delivering this minus the line losses, or whether it means that 500,000 watts are to be delivered at the receiving end of the line. The latter is evidently the correct meaning of the words used.

$\frac{500,000}{5,000}$ = 100 amperes in the line.

$500,000 \times .2 = 100,000$ watts loss in the line.

$\frac{100,000}{100^2}$ = 10 ohms resistance of the line.

Feet in line $= 2 \times 10 \times 5,280 = 105,600$.

$\frac{10}{105,600} = .000095$ = resistance per foot, which corresponds nearly to number o B and S wire, weighing .3199 lbs. per foot.

$3199 \times 105\ 600 = 33{,}780$ lbs. weight of wire.

Example 4.

What would e the weight of wire in the last example, if the voltage were increased to 10,000 and 20,000 volts?

Solution:

$(\frac{5{,}000}{10{,}000})^2 = \frac{1}{4}$ and pounds of copper $= \frac{1}{4}$ 33,780 $= 8{,}445$.

$(\frac{5{,}000}{20{,}000})^2 = \frac{1}{16}$ and pounds of copper $= \frac{1}{16}$ 33,780 $= 2{,}111$.

Plant Efficiency.

Plant efficiency is the ratio of the power delivered to the power generated. In estimating the efficiency of a plant, it must always be distinctly understood just what losses are to be taken into consideration. Much confusion has arisen by neglecting this precaution, and the invention of such terms as mechanical efficiency and electrical efficiency has not tended to lessen this confusion. In estimating the efficiency, the losses are usually taken from the dynamo pulley to the lamp terminals for lighting, or to the motor pulley for power. The method of procedure depends on whether we start from the generating or receiving end.

Note.—Efficiency is nearly always given as a percentage. In speaking of multiplying or dividing by efficiency, it is

always understood that the efficiency, if given as a percentage, must first be divided by 100.

Rule 74. To obtain the energy delivered by any apparatus, multiply the energy delivered to it, by the efficiency

or vice versa,

To obtain the energy delivered to any apparatus, divide the energy delivered by it, by the efficiency.

EXAMPLE 5.

The efficiency of a plant transmitting 480,000 watts is 65%. How much energy must be generated:

Solution:

$\frac{480,000}{.65}$=740,000 watts.

Rule 75. To find the efficiency of a plant containing several sources of loss of energy in series, the efficiencies of each of the pieces of apparatus are multiplied together.

EXAMPLE 6.

Dynamo 93% efficiency, line 80%, motor 85%. What is the efficiency from dynamo pulley to motor pulley?

Solution?

Efficiency of plant=.93×.80×.85=63¼%.

EXAMPLE 7.

The system consists of a dynamo of 95% efficiency, step up transformer (a transformer which transforms from a low voltage to a high) 97%, line 85%, step down transformers 94%, energy delivered by the step down transformers 360 kilowatts. What is the efficiency of the system and the kilowatts delivered to each part?

Solution:

Efficiency of the system $.95\times.97\times.85\times.94=73\frac{1}{2}\%$.

Amount delivered to the step down transformers $\frac{360}{.94}=383$.

Amount delivered to the line $\frac{383}{.85}=451$ kilowatts.

Amount delivered to the step down transformers $\frac{451}{.97}=465$.

Amount delivered to the dynamo $\frac{465}{.95}=490$.

Plant efficiency again $=\frac{360}{490}=73\frac{1}{2}\%$ nearly.

The efficiencies treated so far are the full load efficiencies. That is the efficiency when the apparatus is working at its full rated capacity. In most apparatus the efficiency is less as the load becomes less, for the reason that some of the losses remain constant at all loads. The efficiency of a line, however, increases as the load falls off, for at half load, for instance, the current being halved the loss in the line is only one-quarter and in some cases would be even less if the voltage rises as the load comes off, thus reducing the current required.

APPENDIX A.

ELECTRIC RAILWAYS.

MOTORS.

The rules applying to direct current motors given in Chapter VI, apply equally as well to street car motors and they will therefore not be repeated here.

In stationary motors the most prominent factor to be considered, is the work the motor is capable of delivering without getting too hot and without slowing down too much. In street car motors the most prominent factor is the torque (or better the tractive force) the motor is able to develop at various speeds and loads.

The tractive force is the force exerted by the motor on the car in the direction of its motion.

When a car is moving at an uniform rate of speed, we may consider that the tractive force required to propel the car at that rate of speed exactly equals the tractive force developed by the motor.

We will first consider the conditions which affect the tractive force required to drive the car.

The tractive force required on a level varies with the condition of track and roadbed, lubrication of journals, construction of cars and trucks, weight,

grade and so forth, but other things being equal, we have:

Rule 76. The tractive force varies directly as the weight of the car plus its passengers.

Thus a tractive force of 25 pounds per ton is ordinarily assumed for a car on a level, with track in good condition, running at a speed of 6 to 10 miles an hour. An 8-ton car will therefore require $25 \times 8 = 200$ pounds tractive effort on a level.

Rule 77. To find the watts required to propel a car at a certain speed on a level, multiply the tractive force by the speed in feet per minute, and divide by 33,000 to get the horse-power. Multiply this by 746 to get the watts output of the motor, divide this by the efficiency of the motor to get the watts required by the motor.

EXAMPLE 1.

How many watts are supplied to the motors of a street car weighing 10 tons, tractive force per ton 25 pounds, running at 10 miles an hour on a level. Efficiency of motor 70%.

Solution:

Tractive force $10 \times 25 = 250$.

Feet per minute $\frac{10 \times 5,280}{60} = 880$.

Foot-pounds $= 880 \times 250 = 220,000$.

Horse-power $= \frac{220,000}{33,000} = 6.4$ nearly.

Watts $= 6.4 \times 746 = 4,775$.

Watts supplied to motors $= \frac{4,775}{.70} = 6,820$ nearly.

When the car is climbing a grade a certain amount of energy is required to draw it in addition to what is required on a level. This is represented by the amount of energy required to raise the car through the distance it travels vertically.

Rule 78. To find the energy required for a car to overcome a grade in addition to what would be required to drive it on a level track, first find the number of feet it is raised vertically per minute by multiplying its rate of speed in feet per minute, by the percentage of grade divided by 100. Then multiply this by the weight of the car in pounds, and divide by 33,000 to get the horse-power. Then multiply the horse-power by 746 to get the watts output, and divide this by the efficiency of the motor to get the input.

EXAMPLE 2.

How many watts must be supplied to the motors of the car in the last example to enable it to climb a 12% grade at 10 miles an hour?

Solution:

Vertical rise in feet per minute $880 \times .12 = 105.6$.
Weight of car in pounds $10 \times 2{,}000 = 20{,}000$.
Foot-pounds $= 20{,}000 \times 105.6 = 2{,}112{,}000$.
Horse-power $= \frac{2{,}112{,}000}{33{,}000} = 64$.
Watts $= 64 \times 746 = 47{,}744$.

Total watts output of motors==47,700+6,800= 54,500.

Total watts input of motors=$\frac{54,500}{70}$=77,860.

In practice street car motors are not built to climb grades of 12% at the rate of 10 miles an hour, for as this example shows, a car which would not require more than 7 kilowatts motor capacity at this speed on a level, would require more than 10 times that much on a grade, and since such grades are rare and seldom very long it is cheaper to let the car slow down considerably rather than to run the motor very much below its capacity on a level.

On account of the comparatively low speeds at which street cars run as compared with steam cars, and on account of the inferior condition of track and roadbed, no very reliable data have been obtained as to the variation of tractive force with speed or as to the effect of switches, curves and so forth.

The way in which the motor supplies the necessary tractive force is this : When the load on the motor is increased either by taking on of more passengers or by an increase in the grade, the motor slows down slightly thus reducing its counter E. M. F. and allowing more current to pass through the armature coils and field coils which are in series with the armature coils. Thus the field strength is increased as well as the armature current and since the torque of the motor varies directly as

the field strength multiplied by the armature current (the magnetic reaction of the armature being disregarded) the torque is increased also. This increase of torque increases with the slowing down of the motor until it stops, when the torque is a maximum. As shown, however, in Chapter VI, the energy developed by the motor is a maximum when the counter E. M. F. is one-half the applied, the energy falling off after that because the speed falls faster than the torque increases.

THE CONTROLLING MECHANISM.

The speed of street car motors is regulated in various ways. One very common means employed is to insert a resistance in series with the motor thus reducing the E. M. F. applied at its terminals. This cuts down the current thus reducing the speed.

Another means is by dividing the field coils up into sections and arranging these sections in various ways. The number of sections is usually 3 and at full speed they are placed all in multiple, then through various combinations they are arranged all in series, which increases the resistance in series and also the field strength, both of these things having a tendency to cut down the speed.

This method is more efficient than the use of a simple external resistance, but it is somewhat more complicated, and it is harder to get sufficient radiating surface to dispose of the heat.

Another device for varying the speed is by cutting out a field coil altogether. This is the same in principle as the last mentioned method.

Still another device where there are 2 motors to a car, is to throw them in series at low speeds and in multiple at high speeds. Thus at low speeds the E. M. F. at the terminals of each motor is reduced to one-half the E. M. F.

Another thing which had to be accomplished by the controller was to start the car smoothly. When the car is standing still, there being no counter E. M. F. there is a tendency for a large rush of current which would start the car with a jerk; this is obviated by placing a high resistance (called the starting rheostat) in series with the motor, which is cut out as soon as the car has gathered speed.

THE OVERHEAD LINE.

The calculation of the amount of copper required in the overhead line depends greatly on the kind of ground return used. Where dependence is placed on rail bonding alone for the ground return, its resistance is apt to be very high. Water courses, gas and water mains and so forth are often used, and it is thus easily seen that it is impossible to predict what the resistance of this part of the circuit will be. Experience is the best teacher for such calculations. After having gotten some-

where near the correct weight of copper by preliminary calculations, it will then be necessary to make an "educated guess" guided by experience.

If the resistance of the line be too high, besides the excessive line loss, the speed of the car falls off at a distance from the power station on account of the excessive drop in the E. M. F. This excessive drop in E. M F. is very plainly shown when a car is started at night at a distance from the power house. The lamps in the car owing to the drop in E. M. F. due to the excessive rush of current at starting, burn very dim until the car gathers headway and thus cuts down the current.

The amount of current required per car varies very much under different conditions and must be calculated by the rules given above in this appendix for finding the energy required. The cars must be spread over the line in what may be called a fair average position, the current required by each calculated, and then the allowable resistance to each car from the station, by the methods given in Chapter VIII.

THE STATION.

The amount of energy the generators at the station must deliver per car on the line, depends on the condition of track, number and distribution of curves, grades and so forth, and also to a large extent on the number of cars. With a small number

of cars the fluctuation in the current required from the station is enormous, and jumps suddenly from very light to very heavy loads. When the number of cars is large on the other hand the load remains much steadier, since if a small number of cars happen to require a large current at the same time the increase would be a small proportion of the total energy required by all the cars. In small plants therefore the station must be designed for a much larger horse-power per car than for a large one.

The loss in the line varies considerably with the different methods of obtaining a ground return, but in the best practice it is not over 10%.

In estimating the power required for the station as well as in estimating the copper required in the line, it is well to consider the cars spaced over the line as they will be run, and try them in several positions which may appear to be the most unfavorable and then determine the energy they will require by the rules above given. Then dividing this by the efficiency of the line will give the energy which the dynamos at the station must deliver.

APPENDIX B.

USEFUL TABLES.

TABLE I.

Resistances in legal ohms of wires of different metals and alloys, one foot long and one thousandth of an inch in diameter.

Silver, annealed	9.05
Silver, hard drawn	9.82
Copper, annealed	9.61
Copper, hard drawn	9.83
German silver	125.89
Aluminum, annealed	17.52
Gold, annealed	12.38
Gold, hard drawn	12.60
Zinc, pressed	33 84
Platinum, annealed	54.47
Iron, annealed	58.44
Lead, pressed	118.05
Mercury	572.1

TABLE II.

Weights of a cubic inch of different materials.

MATERIAL.	WEIGHTS.
Steel shaft	.282
Steel castings	.283
Iron, wrought	.278
Iron, mites cast	.276
Iron, sheet	.282
Iron cast, gray	.260
Copper, rolled	.320
Copper, cast	.316
Brass, rolled	305
Brass, cast	.304
No. 3 metal	.315
No. 7 metal	.309
Lead	.408
Tin	.263
Zinc	.253
Hard rubber	.047
Leatheroid	.045
Vulcanized fibre	.050
Porcelain	.086
Slate	.102
Mica	.098

TABLE III.

Table of weights and measures.

WEIGHTS.

One pound = 453.6 grammes.
One pound = .4536 kilogrammes.
One gramme = .0022 pounds.
One kilogramme = 2.2 pounds.

LENGTHS.

One foot = .308 metres.
One mile = 1609 metres.
One mile = 1.61 kilometres.
One inch = 25.4 millimetres.
One millimetre = one thousandth of a metre.
One metre = 3.28 feet.
One metre = 39.37 inches
One kilometre = .6214 miles ($\frac{5}{8}$ mile nearly.)

AREAS.

One square inch = 6.45 square centimetres.
One square inch = 645 square millimetres.
One square centimetre = .155 square inches.
One square millimetre = .00155 square inches.

VOLUMES.

One cubic inch = 16.387 cubic centimetres.
One cubic inch = 16387 cubic millimetres.

TABLE IV.

Temperature Coefficients.

MATERIAL.	COEFFICIENTS.
Silver	.00382
Copper	.00387
Gold	.00367
Zinc	.00370
Cadmium	.00368
Tin	.00360
Lead	.00387
Arsenic	.00389
Antimony	.00308
Bismuth	.00352
Iron	.00511
German silver	.00044

These results are only to be used for temperatures between 0° C and 100° C.

TABLE V.

Table of electro-chemical equivalents.

Name of Substance.	Atomic Weight.	Chemical Equivalent	Electro-Chemical Equivalent in Milligrames per coulomb.
Hydrogen (H.)	1	1	.0105
Potassium (K.)	39.1	39.1	.4105
Sodium (Na.)	23	23	.2415
Gold (Au.)	196.6	65.5	.6875
Silver (Ag.)	108	108	1.1340
Copper,-ic salts (Cu.)	63	31.5	.2307
Copper,-ous salts (Cu.)	63	63	.6615
Mercury,-ic salts (Hg.)	200	100	1.0500
Mercury,-ous salts(Hg)	200	200	2.1000
Tin,-ic salts (Sn.)	118	29.5	.3097
Tin,-ous salts (Sn.)	118	59	.6195
Iron,-ic salts (Fe.)	56	14	.1470
Iron,-ous salts (Fe.)	56	28	.2940
Nickel (Ni.)	59	29.5	.3097
Zinc (Zn.)	65	32.5	.3412
Lead (Pb.)	207	103.5	1.0867
Oxygen (O.)	16	8.	.0840
Chlorine (Cl.)	35.3	35.5	.3727
Iodine (I.)	127	127	1.3335
Bromine (Br.)	80	80	.8400
Nitrogen (N.)	14	4.3	.0490

TABLE VI.

Table showing weight per foot and resistance per foot of pure copper wire at 20° C. Brown and Sharpe gauge.

Diam.	B. & S. No.	Lbs. per ft.	Ohms per ft.
0.460	0000	.6412	.00004879
0.4096	000	.5084	.00006154
0.3648	00	.4033	.00007758
0.3249	0	.3199	.00009775
0.2893	1	.2556	.0001234
0.2576	2	.2011	.0001556
0.2294	3	.1595	.0001962
0.2043	4	.1265	.0002473
0 1819	5	.1003	.0003120
0.162	6	.07952	.0003934
0.1443	7	.0631	.0004958
0.1285	8	.05004	.0006250
0.1144	9	.03966	.0007886
0.1019	10	.03147	.0009940
0.0907	11	.02493	.001255
0.0808	12	.01979	.001581
0.072	13	.01571	.001991
0.0641	14	.01245	.002513
0.0571	15	.009878	.003167
0.0508	16	.007822	.00400
0.0453	17	.006218	.00503
0.0403	18	.004921	.006357
0.0359	19	.003906	.008006
0.032	20	.003103	.01008
0.0285	21	.002462	.01271
0.0253	22	.001940	.01613
0.0226	23	.001548	.02021
0.0201	24	.001224	.02556
0.0179	25	.0009709	.03222

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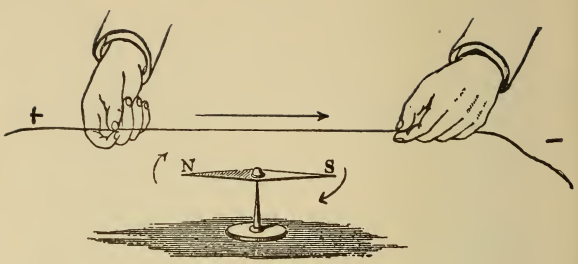

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